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Time-variant redundancy and failure times of deteriorating concrete structures considering multiple limit states

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ABSTRACT

Structural redundancy and load redistribution capacity are desirable features to ensure suitable system performance under accidental actions and extreme events. For deteriorating structures, these features must be evaluated over time to account for the modification of the redistribution mechanisms due to damage processes. In particular, the identification of the local failure modes and prediction of their occurrence in time is necessary in order to maintain a suitable level of system performance and to avoid collapse. In fact, repairable local failures can be considered as a warning of damage propagation and possible occurrence of more severe and not repairable failures. In this paper, failure loads and failure times of concrete structures exposed to corrosion are investigated and life-cycle performance indicators, related to redundancy and elapsed times between sequential failures, are proposed. The effects of the damage process on the structural performance are evaluated based on a methodology for life-cycle assessment of concrete structures exposed to diffusive attack from environmental aggressive agents. The uncertainties involved are taken into account. The proposed approach is illustrated using two applicative examples: a reinforced concrete frame building and a reinforced concrete bridge deck under corrosion. The results demonstrate that both failure loads and failure times can provide relevant information to plan maintenance actions and repair interventions on deteriorating structures in order to ensure suitable levels of structural performance and functionality during their entire life-cycle.

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Introduction

Structural reliability and durability of civil engineering structures and infrastructure facilities are essential to the economic growth and sustainable development of countries. However, aging, fatigue and deterioration processes due to aggressive chemical attacks and other physical damage mechanisms can seriously affect structure and infrastructure systems and lead over time to unsatisfactory structural performance (Clifton & Knab, 1989; Ellingwood, 2005). The economic impact of these processes is extremely relevant (ASCE, 2013; NCHRP, 2006) and emphasises the need of a rational approach to life-cycle design, assessment and maintenance of deteriorating structures under uncertainty based on suitable reliability-based life-cycle performance indicators (Biondini & Frangopol, 2014; Frangopol & Ellingwood, 2010; Saydam & Frangopol, 2011; Zhu & Frangopol, 2012). This need involves a major challenge to the field of structural engineering, since the classical time-invariant structural design criteria and methodologies need to be revised to account for a proper modelling of the structural system over its entire life-cycle by taking into account the effects of deterioration processes, time-variant loadings, and maintenance and repair interventions under uncertainty (Biondini & Frangopol, 2008a, 2016; Frangopol, 2011; Frangopol & Soliman, 2016).

In structural design the level of performance is usually specified with reference to structural reliability. However, when aging and deterioration are considered, the evaluation of the system performance under uncertainty should account for additional probabilistic indicators aimed at providing a comprehensive description of the life-cycle structural resources (Barone & Frangopol, 2014a, 2014b; Biondini & Frangopol, 2014, 2016; Frangopol & Saydam, 2014). The availability of stress redistribution mechanisms and the ability to mitigate the disproportionate effects of sudden damage under accidental actions, abnormal loads and extreme events, are often investigated in terms of structural redundancy (Bertero & Bertero, 1999; Biondini, Frangopol, & Restelli, 2008; Frangopol & Curley, 1987; Frangopol, Iizuka, & Yoshida, 1992; Frangopol & Nakib, 1991; Fu & Frangopol, 1990; Ghosn, Moses, & Frangopol, 2010; Hendawi & Frangopol, 1994; Husain & Tsopelas, 2004; Paliou, Shinozuka, & Chen, 1990; Pandey & Barai, 1997; Schafer & Bajpai, 2005; Zhu & Frangopol, 2013, 2015), structural vulnerability and robustness (Agarwal, Blockley, & Woodman, 2003; Baker, Schubert, & Faber, 2008; Biondini, Frangopol, & Restelli, 2008; Biondini & Restelli, 2008; Ellingwood, 2006; Ellingwood & Dusenberry, 2005; Ghosn et al., 2010; Lind, 1995; Lu, Yu, Woodman, & Blockley, 1999; Starossek & Haberland, 2011), and seismic resilience (Bocchini

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& Frangopol, 2012a, 2012b; Bruneau et al., 2003; Chang & Shinozuka, 2004; Cimellaro, Reinhorn, & Bruneau, 2010; Decò, Bocchini, & Frangopol, 2013).

However, depending on the damage propagation mechanism, continuous deterioration may also involve alternate load redistribution paths and disproportionate effects, which can affect over time structural reliability and other performance indicators, including redundancy, robustness, resilience, and sustainability (Biondini, 2009; Biondini & Frangopol, 2014; Biondini, Camnasio, & Titi, 2015; Biondini, Frangopol, & Restelli, 2008; Biondini & Restelli, 2008; Decò, Frangopol, & Okasha, 2011; Enright & Frangopol, 1999; Frangopol & Bocchini, 2011; Furuta, Kameda, Fukuda, & Frangopol, 2004; Okasha & Frangopol, 2009, 2010; Sabatino, Frangopol, & Dong, 2015; Zhu & Frangopol, 2012, 2013). The effects of continuous deterioration can be particularly relevant for concrete structures exposed to the diffusive attack from aggressive agents, such as chlorides and sulfates, which may involve corrosion of steel reinforcement and deterioration of concrete (Bertolini, Elsener, Pedeferri, & Polder, 2004; CEB, 1992).

For reinforced concrete (RC) structures under corrosion the identification of the local failure modes and of their occurrence in time provides useful information in order to maintain a suitable level of system performance and to avoid collapse over their lifetime. In fact, repairable local failures can be considered as a warning of damage propagation and possible occurrence of more severe and not repairable failures (Mori & Ellingwood, 1994). Structural redundancy is a key performance indicator to this purpose, since it measures the ability of the system to redistribute among its active members the load which can no longer be sustained by other damaged members after the occurrence of a local failure (Biondini, Frangopol, & Restelli, 2008; Frangopol & Curley, 1987; Frangopol et al., 1992). However, this indicator refers to a prescribed point in time and does not provide a direct measure of the failure rate, which depends on the damage scenario and damage propagation mechanism (Biondini & Frangopol, 2008b, 2014).

Failure times and time intervals between subsequent failures, or elapsed time between failures, should be computed to provide complete information about the available resources after occurrence of local failures (Biondini, 2012; Biondini & Frangopol, 2014). In fact, the elapsed time between subsequent failures can be considered as a measure of system redundancy in terms of rapidity of evacuation and/or ability of the system to be repaired right after a critical damage state is reached. More specifically, the identification of all the local failure modes up to collapse and their occurrence in time could be helpful to plan emergency procedures, as well as maintenance and repair interventions to ensure suitable levels of life-cycle system performance and functionality.

In the following, failure loads and failure times of concrete structures under corrosion are investigated. Criteria and methods for the definition of life-cycle performance indicators related to redundancy and elapsed times between subsequent failures are proposed. The effects of the damage process on the structural performance are evaluated by using a proper methodology for life-cycle assessment of concrete structures exposed to diffusive attack from environmental aggressive agents (Biondini, Bontempi, Frangopol, & Malerba, 2004a, 2006). The uncertainties in the material and geometrical properties, in the physical models of deterioration processes, and in the mechanical and environmental stressors, are taken into account in probabilistic terms. The proposed approach is illustrated through the assessment of structural redundancy and elapsed time between failures of a RC frame building and a RC bridge deck under corrosion. The goal is to show that both failure loads and failure times may provide important information to protect, maintain, restore and/or improve the life-cycle structural resources of deteriorating concrete structures.

Time-variant failure loads and failure times

A failure of a system is generally associated with the violation of one or more limit states. Focusing on RC frame systems, limit states of interest may be the occurrence at the material level of local failures associated to cracking of concrete and/or yielding of steel reinforcement, which represent warnings for initiation of damage propagation, as well as attainment of failures associated with the ultimate capacity of critical cross-sections and/or system collapse (Malerba, 1998).

Time-variant failure loads and redundancy

Denoting $\lambda \geq 0$ a scalar load multiplier, the limit states associated to the occurrence of a series of sequential failures $k = 1,2,\ldots$ can be identified by the corresponding failure load multiplier λ_k (Biondini, 2012). Since the structural performance of RC structures deteriorates over time, the functions $\lambda_k = \lambda_k(t)$ need to be evaluated by means of structural analyses taking into account the effects of the damage process (Biondini et al., 2004a).

The ability of the system to redistribute the load after the failure $k = i$ up to the failure $k = j$ depends on the reserve load carrying capacity associated to the failure load multipliers $\lambda_i = \lambda_i(t)$ and $\lambda_j = \lambda_j(t)$:

$$\Delta\lambda_{ij}(t) = \lambda_j(t) - \lambda_i(t) \geq 0 \tag{1}$$

Therefore, the following quantity can be assumed as time-variant measure of redundancy between subsequent failures:

$$\Lambda_{ij}(t) = \Lambda(\lambda_i, \lambda_j) = \frac{\lambda_j(t) - \lambda_i(t)}{\lambda_j(t)} \tag{2}$$

The redundancy factor $\Lambda_{ij} = \Lambda_{ij}(t)$ can assume values in the range [0;1]. It is zero when there is no reserve of load capacity between the failures i and j ($\lambda_i = \lambda_j$), and tends to unity when the failure load capacity λ_i is negligible with respect to λ_j ($\lambda_i << \lambda_j$).

It is worth noting that the classical measure of redundancy refers to the ability of the system to redistribute the load after the occurrence of the first local failure, reached for $\lambda_1 = \lambda_1(t)$, up to structural collapse, reached for a collapse load multiplier $\lambda_c = \lambda_c(t)$. For the sake of brevity, the time-variant redundancy factor between the first failure and collapse is denoted $\Lambda = \Lambda(t)$ (Biondini & Frangopol, 2014):

$$\Lambda(t) = \Lambda(\lambda_1, \lambda_c) = \frac{\lambda_c(t) - \lambda_1(t)}{\lambda_c(t)} \tag{3}$$

It is also noted that redundancy is often associated with the degree of static indeterminacy of the structural system. However, it has been demonstrated that the degree of static indeterminacy is not a consistent measure for structural redundancy (Biondini & Frangopol, 2014; Biondini, Frangopol, & Restelli, 2008; Frangopol & Curley, 1987). In fact, structural redundancy depends on many factors, such as structural topology, member sizes, material properties, applied loads and load sequence, among others (Frangopol & Curley, 1987; Frangopol & Klisinski, 1989; Frangopol & Nakib, 1991). Moreover, the role of these factors may change over time due to structural deterioration, both in deterministic and probabilistic terms (Biondini, 2009; Biondini & Frangopol, 2014; Okasha & Frangopol, 2009).

Failure times and elapsed time between failures

Structural redundancy refers to a prescribed point in time and does not provide information on the failure sequence and failure rate over the structural lifetime. Failure times should be computed to this purpose and the time interval between subsequent failures, or the elapsed time between failures, could represent an effective indicator of the damage tolerance of the system and its ability to be repaired after local failures.

The failure times T_k associated to the occurrence of sequential failures $k = 1,2,\ldots$ can be evaluated by comparing the time-variant failure load multipliers $\lambda_k = \lambda_k(t)$ to prescribed time-variant target functions $\lambda_k^* = \lambda_k^*(t)$ as follows (Biondini, 2012):

$$T_k = \min\left\{\ t \mid \lambda_k(t) < \lambda_k^*(t)\ \right\} \tag{4}$$

After a local failure $k = i$, the ability of the system to delay the failure $k = j$ depends on the elapsed time between these failures occurring at times T_i and T_j (Figure 1):

$$\Delta T_{ij} = T_j - T_i \geq 0 \tag{5}$$

For the sake of brevity, the elapsed time between the first failure, occurring for $\lambda_1 = \lambda_1(t)$ at time T_1, and the structural collapse, reached for $\lambda_c = \lambda_c(t)$ at time T_c, is denoted ΔT (Biondini, 2012; Biondini & Frangopol, 2014):

$$\Delta T = T_c - T_1 \tag{6}$$

This is an important performance indicator, since it provides the residual lifetime after the first damage warning, for example associated with the formation of a plastic hinge, and identifies the time to global failure due to the activation of a set of plastic hinges leading to structural collapse (Biondini & Frangopol, 2008b).

As mentioned previously, the elapsed time between failures can also be considered as a measure of system redundancy in terms of rapidity of evacuation and/or ability of the system to be repaired right after a critical damage. However, even though they are related concepts, elapsed time between failures and structural redundancy refer to different system resources.

Role of the uncertainties

The geometrical and material properties of the structural systems, the mechanical and environmental stressors, and the parameters of the deterioration processes are always uncertain. Consequently, life-cycle prediction models have to be formulated in probabilistic terms and all parameters of the model have to be considered as random variables or processes. Therefore, the time evolution of the failure loads $\lambda_k = \lambda_k(t)$ and the corresponding failure times T_k, with $k = 1,2,\ldots$, are also random variables or processes, as shown in Figure 2 where uncertainties are associated with initial load capacity, damage initiation, deterioration rate, load capacity after maintenance/repair interventions, and failure time without or with maintenance/repair (Frangopol, 2011; Biondini & Frangopol, 2016). Therefore, a lifetime probabilistic analysis is necessary to investigate the time-variant effects of uncertainty on both redundancy factors $\Lambda_{ij} = \Lambda_{ij}(t)$ and elapsed time intervals ΔT_{ij} between the failures i and j.

Deterioration modelling: a review

A life-cycle probabilistic-oriented approach to design and assessment of structural systems must be based on a reliable and effective modelling of structural deterioration mechanisms. Deterioration models could be developed on empirical bases, as it is generally necessary for rate-controlled damage processes, or founded on mathematical descriptions of the underlying physical

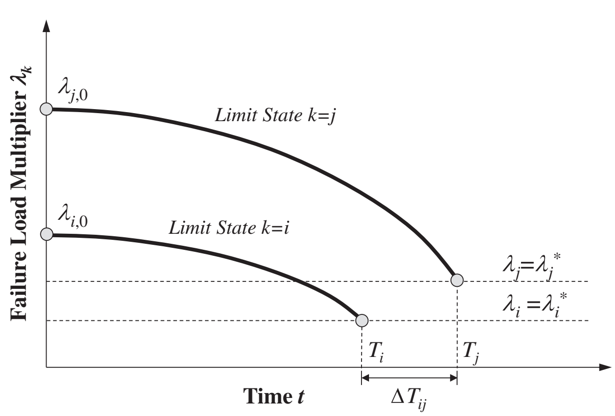

Figure 1. Time-variant failure load multipliers λ_i and λ_j, failure times T_i and T_j associated with the two limit states $\lambda_i = \lambda_i^*$ and $\lambda_j = \lambda_j^*$, and elapsed time ΔT_{ij} between these two sequential failures.

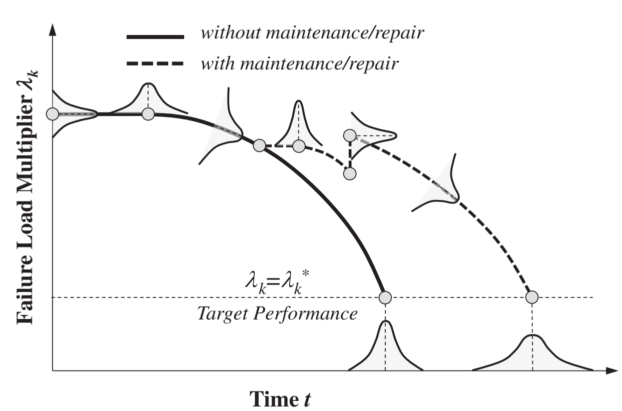

Figure 2. Time-variant failure load multiplier with uncertainties associated with initial load capacity, damage initiation, deterioration rate, load capacity after maintenance and repair interventions, and failure time without or with maintenance/repair.

mechanisms, as it is often feasible for diffusion-controlled damage processes (Ellingwood, 2005).

The latter is the case of interest for RC structures exposed to chloride ingress, where damage induced by the diffusive attack may involve corrosion of steel reinforcement and deterioration of concrete (Bertolini et al., 2004; CEB, 1992). The modelling of these processes should account for both the diffusion process and the related mechanical damage, as well as for the coupling effects of diffusion, damage and structural behaviour.

Diffusion process

The diffusion of chemical components in solids can be described by the Fick's laws which, in the case of a single component diffusion in isotropic, homogeneous and time-invariant media, can be reduced to the following second order linear partial differential equation (Glicksman, 2000):

$$D\nabla^2 C = \frac{\partial C}{\partial t} \tag{7}$$

where D is the diffusivity coefficient of the medium, $C = C(\mathbf{x}, t)$ is the concentration of the chemical component at point $\mathbf{x} = (x, y, z)$ and time t, $\nabla C = \mathbf{grad}\, C(\mathbf{x}, t)$ and $\nabla^2 = \nabla \cdot \nabla$.

For one-dimensional diffusion, the Fick's equation is amenable to be solved analytically. This analytical solution is a convenient mathematical tool for practical applications (fib, 2006). However, the actual diffusion processes in concrete structures are generally characterised by two- or three-dimensional patterns of concentration gradients. For this reason, a numerical solution of the Fick's diffusion equation may be necessary for accurate life-cycle assessment of corroding RC structures (Titi & Biondini, 2016).

In this study, the diffusion equation is solved numerically by using cellular automata (Wolfram, 1994). With reference to a regular uniform grid of cells in two dimensions, the Fick's model can be reproduced at the cross-sectional level by the following evolutionary rule (Biondini et al., 2004a):

$$C_{ij}^{k+1} = \phi_0 C_{ij}^k + \frac{1-\phi_0}{4}\left(C_{i,j-1}^k + C_{i,j+1}^k + C_{i-1,j}^k + C_{i+1,j}^k\right) \tag{8}$$

where the discrete variable $C_{ij}^k = C(\mathbf{x}_{ij}, t_k)$ represents the concentration of the component in the cell (i, j) at point $\mathbf{x}_{ij} = (y_i, z_j)$ and time t_k, and $\phi_0 \in [0;1]$ is a suitable evolutionary coefficient. In particular, to regulate the process according to a given diffusivity D, the grid dimension Δx and the time step Δt of the automaton must satisfy the following relationship:

$$D = \frac{1-\phi_0}{4}\frac{\Delta x^2}{\Delta t} \tag{9}$$

A proof is given in Biondini, Frangopol, and Malerba (2008).

The deterministic value $\phi_0 = 1/2$ usually leads to a good accuracy of the automaton. However, the local stochastic effects in the mass transfer can be taken into account by assuming ϕ_0 as random variable. The stochastic model also allows to simulate the interaction between diffusion process and mechanical behaviour of the damaged structure. Further details can be found in Biondini et al. (2004a).

Corrosion process

The most relevant effect of corrosion in concrete structures is the reduction of the cross-section of the reinforcing steel bars. The area $A_s = A_s(t)$ of a corroded bar can be represented as follows (Biondini et al., 2004a):

$$A_s(t) = [1 - \delta_s(t)]\, A_{s0} \tag{10}$$

where A_{s0} is the area of the undamaged steel bar and $\delta_s = \delta_s(t)$ is a dimensionless damage index which provides a measure of cross-section reduction in the range [0; 1].

Effects of corrosion are not limited to damage of reinforcing steel bars. In fact, the formation of oxidation products may lead to propagation of longitudinal splitting cracks and concrete cover spalling (Al-Harthy, Stewart, & Mullard, 2011; Cabrera, 1996; Vidal, Castel, & François, 2004). In this study, the local deterioration of concrete is modelled by means of a degradation law of the effective resistant area of concrete matrix $A_c = A_c(t)$ (Biondini et al., 2004a):

$$A_c(t) = [1 - \delta_c(t)]\, A_{c0} \tag{11}$$

where A_{c0} is the area of undamaged concrete and $\delta_c = \delta_c(t)$ is a dimensionless damage index which provides a measure of concrete deterioration in the range [0; 1]. However, it may be not straightforward to relate the damage function $\delta_c = \delta_c(t)$ to the amount of steel mass loss. For this reason, in some cases, it could be more convenient to model the concrete deterioration due to splitting cracks and cover spalling through a reduction of the concrete compression strength (see Biondini & Vergani, 2015).

Additional effects of corrosion may occur depending on the type of corrosion mechanisms, i.e. uniform corrosion, localised (pitting) corrosion, or mixed type of corrosion (Stewart, 2009; Zhang, Castel, & François, 2010). As an example, depending on the amount of steel mass loss, non uniform corrosion may involve a remarkable reduction of steel ductility (Almusallam, 2001; Apostolopoulos & Papadakis, 2008) and a limited reduction of steel strength (Du, Clark, & Chan, 2005). Further information for a proper modelling of these effects can be found in Biondini and Vergani (2015). In this study, the effects of uniform corrosion only are investigated.

Damage rates

For diffusion-controlled damage processes, the deterioration rate depends on the time-variant concentration of the diffusive chemical components. In such processes, damage induced by mechanical loading interacts with the environmental factors and accelerates both diffusion and deterioration. Therefore, the dependence of the deterioration rate on the concentration of the diffusive agent is generally complex, and the available information about environmental factors and material characteristics is usually not sufficient for a detailed modelling. Despite the complexity of the problem at the microscopic level, simple coupling models can often be successfully adopted at the macroscopic level in order to reliably predict the time evolution of structural performance (Biondini & Frangopol, 2008b; Biondini, Frangopol, & Malerba, 2008; Biondini et al., 2004a).

Based on available data for sulfate and chloride attacks (Pastore & Pedeferri, 1994) and correlation between chloride content and corrosion current density in concrete (Bertolini et al., 2004; Liu & Weyers, 1998; Thoft-Christensen, 1998), a linear relationship between rate of corrosion in the range 0–200 μm/year and chloride content in the range 0–3% by weight of cement could be reasonable for structures exposed to severe environmental conditions. In this study, the time-variant damage indices $\delta_c = \delta_c(\mathbf{x}, t)$ and $\delta_s = \delta_s(\mathbf{x}, t)$ are related to the diffusion process by assuming a linear relationship between the rate of damage and the mass concentration $C = C(\mathbf{x}, t)$ of the aggressive agent:

$$\frac{\partial \delta_c(\mathbf{x}, t)}{\partial t} = \frac{C(\mathbf{x}, t)}{C_c \Delta t_c} = q_c C(\mathbf{x}, t) \tag{12}$$

$$\frac{\partial \delta_s(\mathbf{x}, t)}{\partial t} = \frac{C(\mathbf{x}, t)}{C_s \Delta t_s} = q_s C(\mathbf{x}, t) \tag{13}$$

where C_c and C_s are the values of constant concentration leading to a complete damage of the materials after the time periods Δt_c and Δt_s, respectively. The damage rate coefficients $q_c = (C_c \Delta t_c)^{-1}$ and $q_s = (C_s \Delta t_s)^{-1}$ depend on both the type of corrosion mechanism and corrosion penetration rate. The initial conditions $\delta_c(\mathbf{x}, t_{cr}) = \delta_s(\mathbf{x}, t_{cr}) = 0$ with $t_{cr} = \min\{t \mid C(\mathbf{x}, t) \geq C_{cr}\}$ are assumed, where C_{cr} is a critical threshold of concentration (Biondini et al., 2004a).

Structural analysis considering time effects

The lifetime structural performance is evaluated by means of structural analysis considering time-variant parameters (Biondini & Vergani, 2015; Biondini et al., 2004a). The formulation is based on the general criteria and methods for nonlinear analysis of concrete structures (Malerba, 1998). At cross-sectional level, the vector of the stress resultants (axial force N and bending moments M_z and M_y):

$$\mathbf{r} = \mathbf{r}(t) = \left[N \; M_z \; M_y\right]^T \tag{14}$$

and the vector of the global strains (axial elongation ε_0 and bending curvatures χ_z and χ_y):

$$\mathbf{e} = \mathbf{e}(t) = [\varepsilon_0 \; \chi_z \; \chi_y]^T \tag{15}$$

are related, at each time instant t, as follows:

$$\mathbf{r}(t) = \mathbf{H}(t)\, \mathbf{e}(t) \tag{16}$$

The time-variant stiffness matrix $\mathbf{H} = \mathbf{H}(t)$ of the RC cross-section under corrosion is derived by integration over the composite area of the materials, or by assembling the contributions of both concrete $\mathbf{H}_c = \mathbf{H}_c(t)$ and steel $\mathbf{H}_s = \mathbf{H}_s(t)$:

$$\mathbf{H}(t) = \mathbf{H}_c(t) + \mathbf{H}_s(t) \tag{17}$$

$$\mathbf{H}_c(t) = \int_{A_c(x)} E_c(y, z, t)\, \mathbf{B}(y, z)\, [1 - \delta_c(y, z, t)]\, \mathrm{d}A \tag{18}$$

$$\mathbf{H}_s(t) = \sum\nolimits_m E_{sm}(t)\, \mathbf{B}_m [1 - \delta_{sm}(t)]\, A_{sm} \tag{19}$$

where the symbol m refers to the m^{th} reinforcing bar located at point (y_m, z_m) in the centroidal principal reference system (y, z) of the cross-section, $E_c = E_c(y, z, t)$ and $E_{sm} = E_{sm}(t)$ are the moduli of the materials, $\mathbf{B}(y, z) = \mathbf{b}(y, z)^T\, \mathbf{b}(y, z)$ is a linear operator matrix, and $\mathbf{b}(y, z) = [1 \; -y \; z]$.

It is worth noting that the vectors $\mathbf{r}$ and $\mathbf{e}$ have to be considered as total or incremental quantities based on the nature of the stiffness matrix $\mathbf{H}$, which depends on the type of formulation adopted (i.e. secant or tangent) for the generalised moduli of the materials associated with the stress–strain nonlinear constitutive laws.

The proposed cross-sectional formulation can be extended to formulate the characteristics of RC beam finite elements for time-variant nonlinear and limit analysis of concrete structures under corrosion. Details can be found in Biondini et al. (2004a), Biondini & Frangopol (2008b), Biondini and Vergani (2015).

Applications

The proposed approach is applied to the probabilistic assessment of structural redundancy and elapsed time between failures of a RC frame and a RC bridge deck under corrosion.

Constitutive laws of the materials

The constitutive behaviour of the materials is described in terms of stress–strain nonlinear relationships. For concrete, the Saenz's law in compression and a bilinear elastic-plastic model in tension are assumed, with: compression strength f_c; tension strength $f_{ct} = .25\, f_c^{2/3}$; initial modulus $E_{c0} = 9500\, f_c^{1/3}$; peak strain in compression $\varepsilon_{c0} = .20\%$; strain limit in compression $\varepsilon_{cu} = .35\%$; strain limit in tension $\varepsilon_{ctu} = 2 f_{ct}/E_{c0}$. For steel, a bilinear elastic-plastic model in both tension and compression is assumed, with yielding strength f_{sy}, elastic modulus $E_s = 210$ GPa, and strain limit $\varepsilon_{su} = 1.00\%$ associated with bond failure. In this way, the constitutive laws are completely defined by the material strengths f_c and f_{sy}.

Probabilistic modelling

The probabilistic analysis accounts for the uncertainty in both the geometrical and mechanical characteristics of the structural systems and in the parameters which define the deterioration processes. At cross-sectional level, the probabilistic model of the mechanical behaviour, diffusion process and damage propagation mechanism considers as random variables the material strengths f_c and f_{sy} of concrete and steel, respectively, the coordinates (y_p, z_p) of each nodal point $p = 1,2,\ldots,$ of the cross-section, the coordinates (y_m, z_m) and diameter $\varnothing_m$ of each steel bar $m = 1,2,\ldots,$ the diffusivity coefficient D, and the damage rate coefficients q_c and q_s. Nominal values are assumed as mean values. The probabilistic distributions and coefficients of variation are listed in Table 1 (Biondini et al., 2006; Sudret, 2008; Vismann & Zilch, 1995). The input random variables are uncorrelated to emphasise the effects of the lack of correlation on the investigated output random variables (Harr, 1996). Moreover, high values of the coefficient of

Table 1. Probability distributions and coefficients of variation (nominal values are assumed as mean values μ).

Random variable ($t = 0$)	Type	C.o.V.
Concrete strength, f_c	Lognormal	5 MPa/μ
Steel strength, f_{sy}	Lognormal	30 MPa/μ
Coordinates of nodal points, (y_p, z_p)	Normal	5 mm/μ
Coordinates of steel bars, (y_m, z_m)	Normal	5 mm/μ
Diameter of steel bars, $\varnothing_m$	Normal*	.10
Diffusivity, D	Normal*	.10
Concrete damage rate, $q_c = (C_c \Delta t_c)^{-1}$	Normal*	.30
Steel damage rate, $q_s = (C_s \Delta t_s)^{-1}$	Normal*	.30

*Truncated distributions with non negative outcomes.

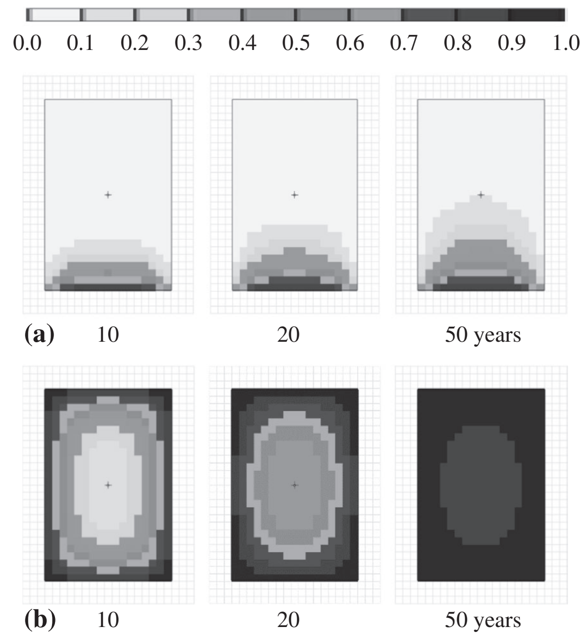

Figure 4. Maps of concentration $C(\mathbf{x}, t)/C_0$ of the aggressive agent after 10, 20, and 50 years from the initial time of diffusion penetration (nominal frame system): (a) scenario (I) with exposure on one side; (b) scenario (II) with exposure on four sides.

variation are adopted for the random variables which mainly influence the time-variant uncertainty of the corrosion damage, such as the steel bar diameter and damage rates.

The lifetime probabilistic analysis is carried out by Monte Carlo simulation. The required accuracy of the simulation process is achieved through a posteriori estimation of the goodness of the sample size based on a monitoring of the time-variant statistical parameters of the random variables under investigation.

RC frame

The lifetime structural performance of the RC frame shown in Figure 3 is investigated in terms of redundancy and elapsed time between failures. The nominal material strengths are $f_c = 40$ MPa for concrete in compression and $f_{sy} = 500$ MPa for the yield of reinforcing steel. The frame is subjected to a dead load $q = 32$ kN/m applied on the beam and a live load λF acting at top of the columns, with $F = 100$ kN.

The frame system is designed with cross-sectional stiffness and bending strength capacities much larger in the beam than in the columns. Moreover, shear failures are avoided over the lifetime by a proper capacity design (Celarec, Vamvatsikos, & Dolšek, 2011; Titi & Biondini, 2014). In this way, a shear-type behaviour can be assumed, with the critical regions where plastic hinges are expected to occur located at the ends of the columns.

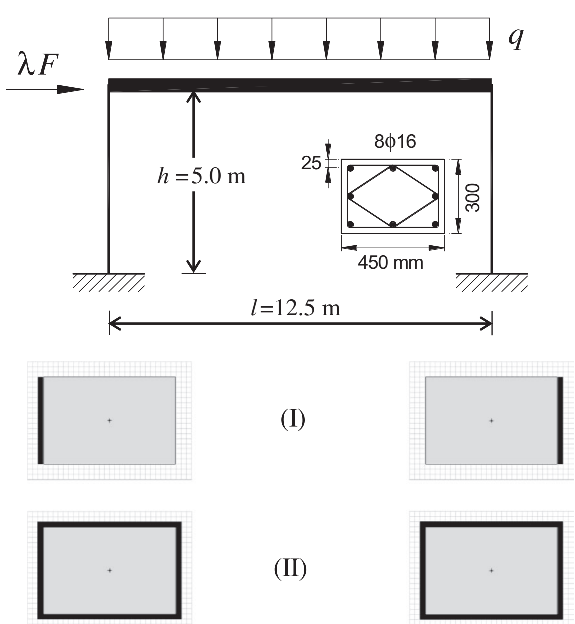

Figure 3. Reinforced concrete frame: geometry, structural scheme, cross-section of the columns, loading condition, grid of the diffusion model, and exposure scenarios with (I) columns exposed on one side and (II) columns exposed on four sides.

The structure is subjected to the diffusive attack from an aggressive agent located on the external surfaces of the columns with concentration C_0. The two exposure scenarios shown in Figure 3 are considered, with (I) columns exposed on the outermost side only, or (II) columns exposed on the four sides. A nominal diffusivity coefficient $D = 10^{-11}$ m^2/sec is assumed. Figure 4 shows the deterministic maps of concentration $C(\mathbf{x}, t)/C_0$ for the two investigated exposure scenarios after 10, 20, and 50 years from the initial time of diffusion penetration.

The corrosion damage induced by diffusion is evaluated by taking the stochastic effects in the mass transfer into account (Biondini et al., 2004a). Corrosion of steel bars with no deterioration of concrete is assumed, with nominal damage parameters $C_s = C_0$, $\Delta t_s = 50$ years, and $C_{cr} = 0$. This model reproduces a deterioration process with severe corrosion of steel, as may occur for carbonated or heavily chloride-contaminated concrete and high relative humidity, conditions under which the corrosion rate can reach values above 100 μm/year (Bertolini et al., 2004).

Figure 5 shows the evolution over a 50-year lifetime of the failure load multipliers $\lambda_1 = \lambda_1(t)$ and $\lambda_c = \lambda_c(t)$ associated to the reaching of first local yielding of steel reinforcement and structural collapse of the frame system, respectively. The failure loads are computed at each time instant under the hypotheses of linear elastic behaviour up to first local yielding, and perfect plasticity at collapse.

The time evolution of the redundancy factor $\Lambda = \Lambda(t)$ of the frame system for the two investigated scenarios is shown in Figure 6. It is noted that for case (I) redundancy increases over time, even if the structural performance in terms of load capacity decreases. This is because the bending strength of the critical cross-sections corroded on the compression side deteriorates

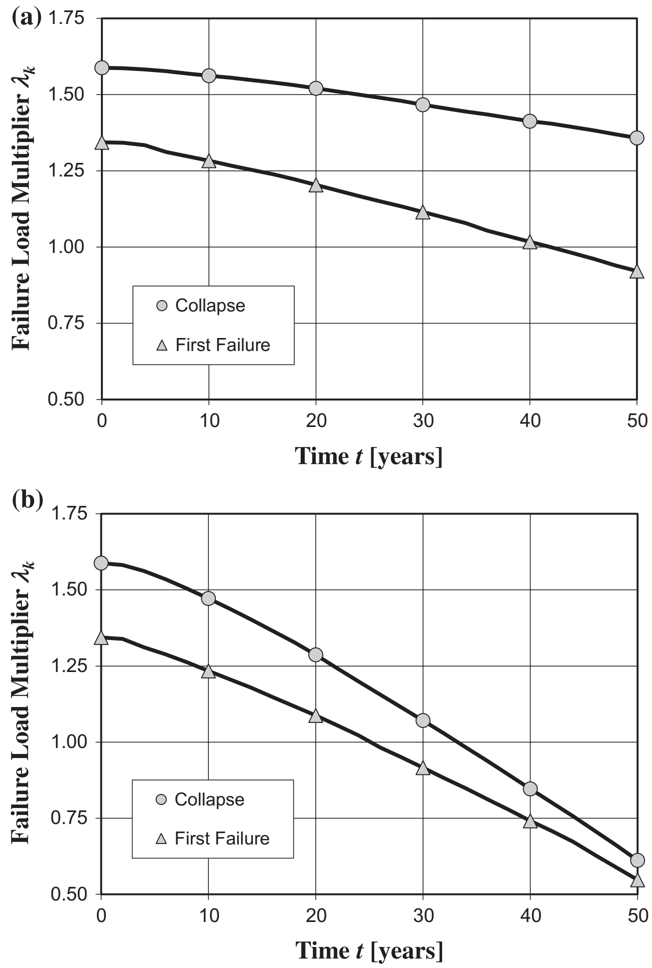

Figure 5. Time evolution of the load multipliers λ_1 and λ_c associated with the reaching of first local yielding of steel reinforcement and collapse, respectively: (a) scenario (I) with exposure on one side; (b) scenario (II) with exposure on all sides.

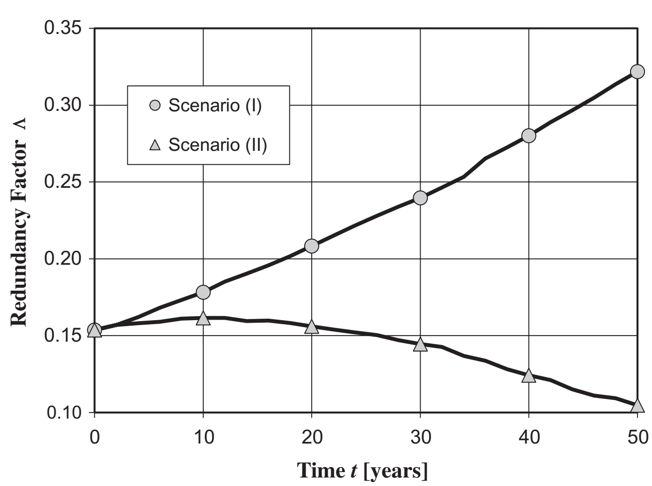

Figure 6. Time evolution of the redundancy factor Λ for scenario (I) with exposure on one side, and scenario (II) with exposure on all sides.

more slowly compared to the cross-sections corroded on the tension side. Therefore, the collapse load multiplier λ_c, which depends on the bending strengths of all critical cross-sections, has a lower deterioration rate than the load multiplier at first yielding λ_1, which is associated with the failure of a single

Table 2. Failure times T_1 and T_c and elapsed time ΔT associated to different target load values λ^*.

λ^*	T_1 [years]	T_c [years]	ΔT [years]
.70	41.4	46.4	5.0
.80	36.2	42.1	5.9
.90	30.8	37.7	6.9
1.00	25.1	33.2	8.1
1.10	19.2	28.6	9.4
1.20	12.4	24.0	11.6
1.30	4.9	19.3	14.5

cross-section. On the contrary, redundancy mainly decreases over time for case (II). Therefore, case (II) is the worst damage scenario for structural redundancy.

The time evolution of the failure loads (Figure 5) indicates that the exposure scenario (II) may be critical with respect to structural collapse. Therefore, for this scenario it is of interest the assessment of the failure times T_1 and T_c associated to the occurrence of the first yielding and collapse, respectively, as well as the elapsed time $\Delta T = T_c - T_1$ between such failures. Failure times and elapsed times associated to different target values $\lambda^* = \lambda_1^* = \lambda_c^*$ are listed in Table 2 for the nominal system. These values indicate that after local failures a significant rapidity of repair may be required under severe exposures. Moreover, it can be noticed that the failure times decrease as the target load multiplier increases. The elapsed time between failures shows instead an opposite trend. Therefore, the availability of a larger reserve of load capacity with respect to the design target is beneficial to delay the occurrence of failures, but it may require prompter repair actions after local failures occur.

The effects of the uncertainty are investigated based on the probabilistic information given in Table 1. The two sets of random variables associated to each column are preliminarily assumed as uncorrelated to emphasise the effects of the uncertainty. Figure 7 shows the probability mass functions (PMFs) of the failure times T_1 and T_c (Figure 7(a)) and elapsed time ΔT (Figure 7(b)) for two deterministic values of the target load multiplier, $\lambda^* = 1.00$ and $\lambda^* = .75$, based on a sample of 2000 Monte Carlo realisations. For $\lambda^* = 1.00$ the failure times T_1 and T_c are characterised by mean and standard deviation values lower than the values obtained for $\lambda^* = .75$. On the contrary, the mean value of the elapsed time ΔT is higher for $\lambda^* = 1.00$ than for $\lambda^* = .75$, with a small difference in terms of dispersion. These results confirm that a suitable reserve of load capacity with respect to the design target allows to delay the possible occurrence of failure events, but it demands for higher promptness and rapidity in the recovery actions.

It is worth noting that the effects of randomness on the reserve load capacity $\Delta\lambda = \lambda_c - \lambda_1$ lead to mean values of the elapsed time ΔT higher than the nominal deterministic values. Moreover, the strong correlation between the failure load multipliers λ_1 and λ_c is beneficial to achieve a lower variance of the elapsed time ΔT than the variance of the failure times T_1 or T_c.

The influence of correlation is also studied by assuming the two sets of random variables associated to each column as fully correlated. The results lead to conclusions similar to the case of uncorrelated variables, with small changes in the probabilistic parameters of the investigated performance indicators. As an example, the mean and standard deviation values obtained for the elapsed time ΔT are $\mu = 11.1$ years and $\sigma = 4.2$ years for $\lambda^* = 1.00$, and $\mu = 7.3$ years and $\sigma = 2.6$ years for $\lambda^* = .75$.

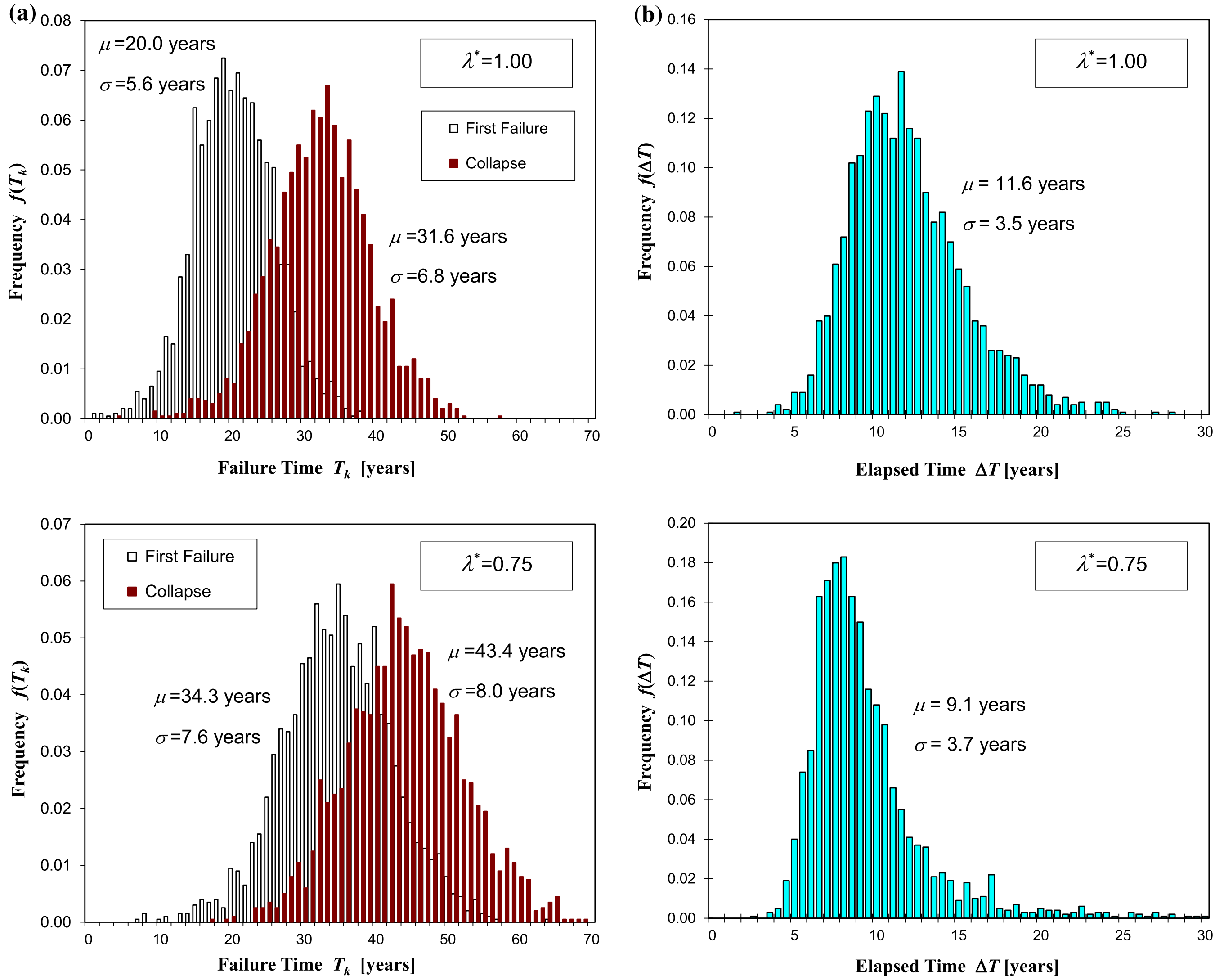

Figure 7. PMFs of (a) failure times T_1 and T_c and (b) elapsed time between failures ΔT for two values of the target load multiplier, $\lambda^* = 1.00$ and $\lambda^* = .75$, under scenario (II) with exposure on all sides.

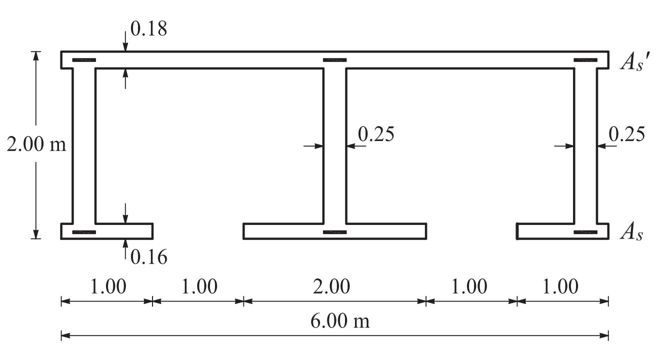

Figure 8. Reinforced concrete bridge deck: geometry of the cross-section and main steel reinforcement $A_s' = 48\varnothing28$ mm and $A_s = 21\varnothing28$ mm (additional reinforcing steel bars: $130\varnothing8$ mm in the top slab; $60\varnothing8$ mm in the bottom slab).

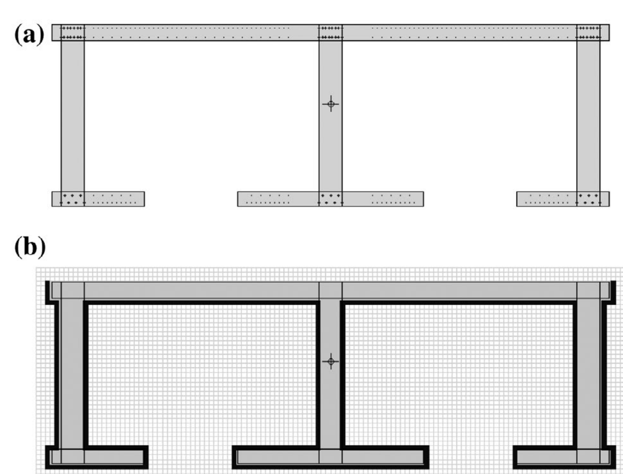

Figure 9. (a) Structural model of the bridge deck cross-section with indication of the steel reinforcement (259 steel bars: $48\varnothing28$ mm + $130\varnothing8$ mm in the top slab; $21\varnothing28$ mm + $60\varnothing8$ mm in the bottom slab). (b) Grid of the diffusion model and exposure scenario.

RC bridge deck

The lifetime structural performance of a RC bridge deck is investigated at cross-sectional level in terms of redundancy and elapsed time between failures. The geometry of the concrete cross-section

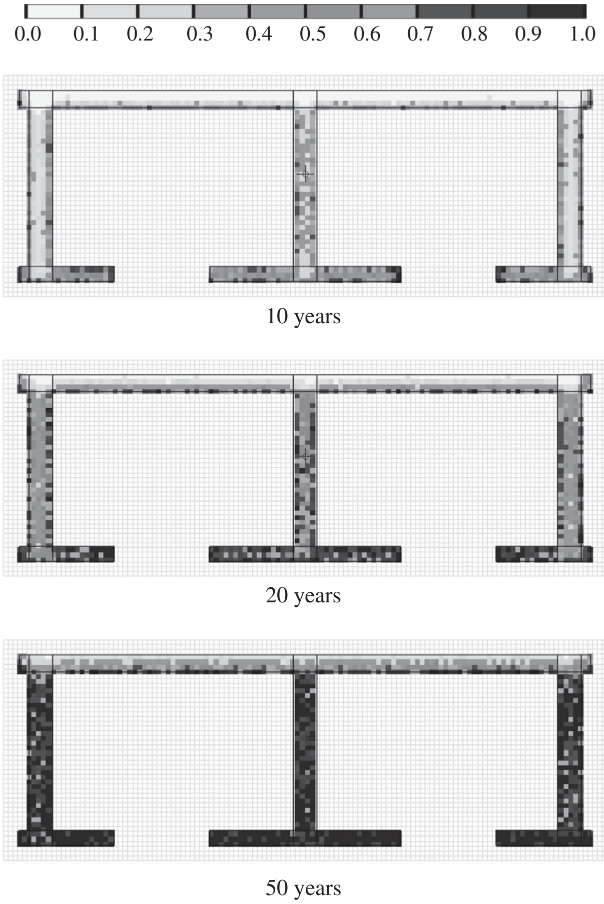

Figure 10. Maps of concentration $C(\mathbf{x}, t)/C_0$ of the aggressive agent after 10, 20, and 50 years from the initial time of diffusion penetration (nominal bridge deck under stochastic diffusion).

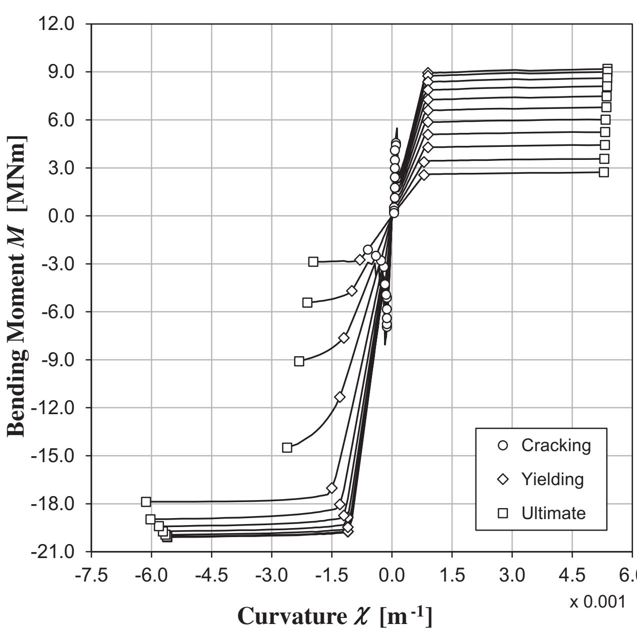

Figure 11. Time evolution of the nominal bending moment M vs. curvature χ diagrams over a 50-year lifetime (Δt = 5 years), with indication of the points associated to first cracking of concrete, first yielding of steel reinforcement, and ultimate flexural capacity of the cross-section.

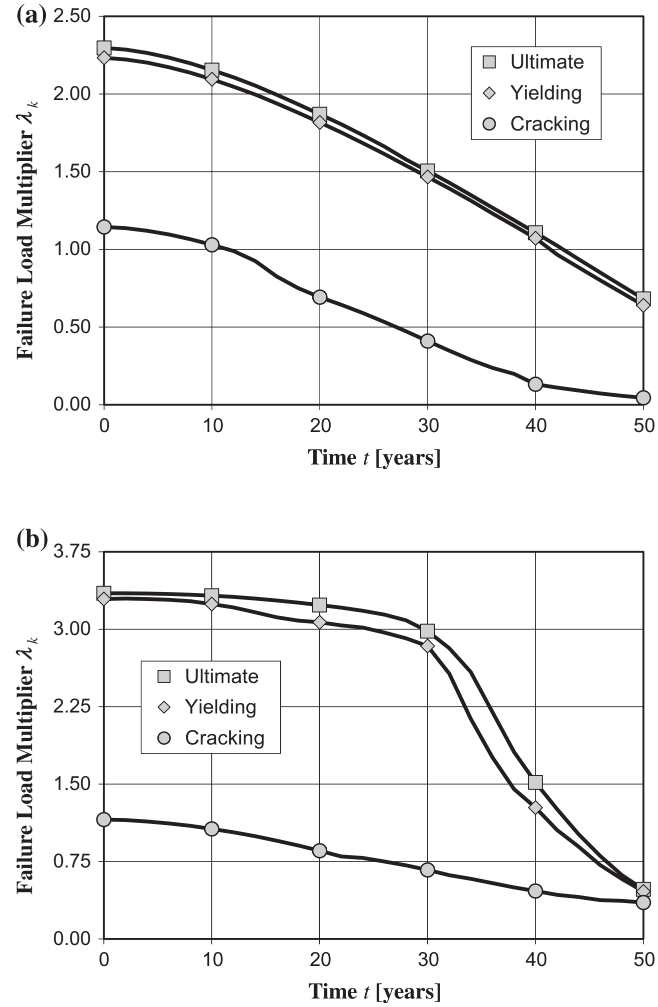

Figure 12. Time evolution of the failure load multipliers λ_k, with k = 1,2,3, at concrete first cracking (k = 1), steel first yielding (k = 2), and cross-section ultimate capacity (k = 3): (a) positive and (b) negative bending moment.

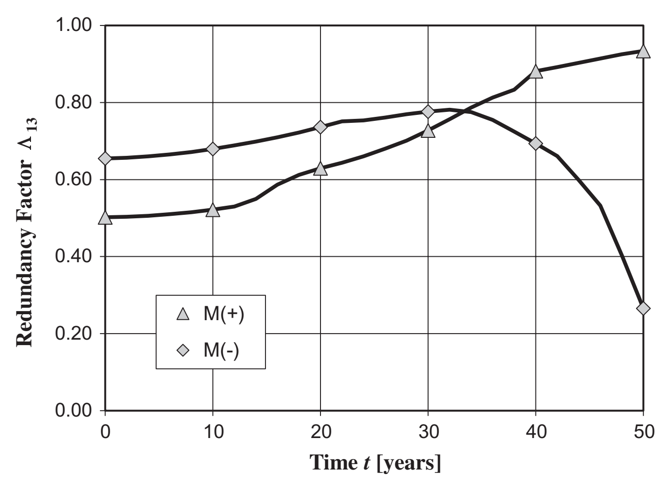

Figure 13. Time evolution of the nominal redundancy factor Λ_{13} between the states (1) and (3) associated to first concrete cracking and cross-section ultimate capacity, respectively, for positive and negative bending moment.

Table 3. Failure times T_1, T_2, T_3, and elapsed times between failures ΔT_{12}, ΔT_{23} [years]: (1) concrete first cracking; (2) steel first yielding and (3) cross-section ultimate capacity.

$\lambda^* = 1.0$	T_1	T_2	T_3	ΔT_{12}	ΔT_{23}
M^+	11.5	41.4	42.6	29.9	1.2
M^-	13.7	42.6	44.1	28.9	1.5

and the location of the main steel reinforcement in the top and bottom slabs are shown in Figure 8. The nominal dimensions are: width = 6.00 m; depth = 2.00 m; web thickness = .25 m; top slab thickness = .18 m; bottom slab thickness = .16 m. The steel reinforcement located in the top slab consists of 48 bars with nominal diameter $\varnothing$ = 28 mm, and 130 bars with $\varnothing$ = 8 mm. The steel reinforcement located in the bottom slab consists of 21 bars with $\varnothing$ = 28 mm and 60 bars with $\varnothing$ = 8 mm. Figure 9(a) shows the structural model of the cross-section, with detailed location of the steel bars. The nominal material strengths are f_c = 30 MPa for concrete in compression and f_{sy} = 300 MPa for the yield of reinforcing steel.

The bridge deck cross-section is subjected to the diffusive attack from an aggressive agent located with concentration C_0 on the external surface exposed to the atmosphere. The diffusion model and the exposure scenario are shown Figure 9(b). A nominal diffusivity coefficient $D = 10^{-11}$ m²/sec is assumed. Figure 10 shows the stochastic maps of concentration $C(\mathbf{x}, t)/C_0$ after 10, 20, and 50 years from the initial time of diffusion penetration.

A severe corrosion damage scenario is assumed, with nominal parameters $C_c = C_s = C_0$, $\Delta t_c = 25$ years, $\Delta t_s = 50$ years, and $C_{cr} = 0$. The mechanical damage induced by diffusion over a 50-year lifetime is shown in Figure 11 in terms of nominal bending moment M vs. curvature χ diagrams computed by assuming the bridge deck axially unloaded.

The results shown in Figure 11 indicate that damage significantly affect the flexural performance of the cross-section, both for positive and negative bending moments. Deterioration of structural performance is mainly due to the severe exposure of the bottom slab. For positive bending moment, the corrosion of the reinforcing steel bars in tension located in the bottom slab leads to a progressive bending strength deterioration over the lifetime, with no significant changes in the curvature ductility. For negative bending moment, the lower corrosion rate of the reinforcing steel bars in tension located in the top slab involves

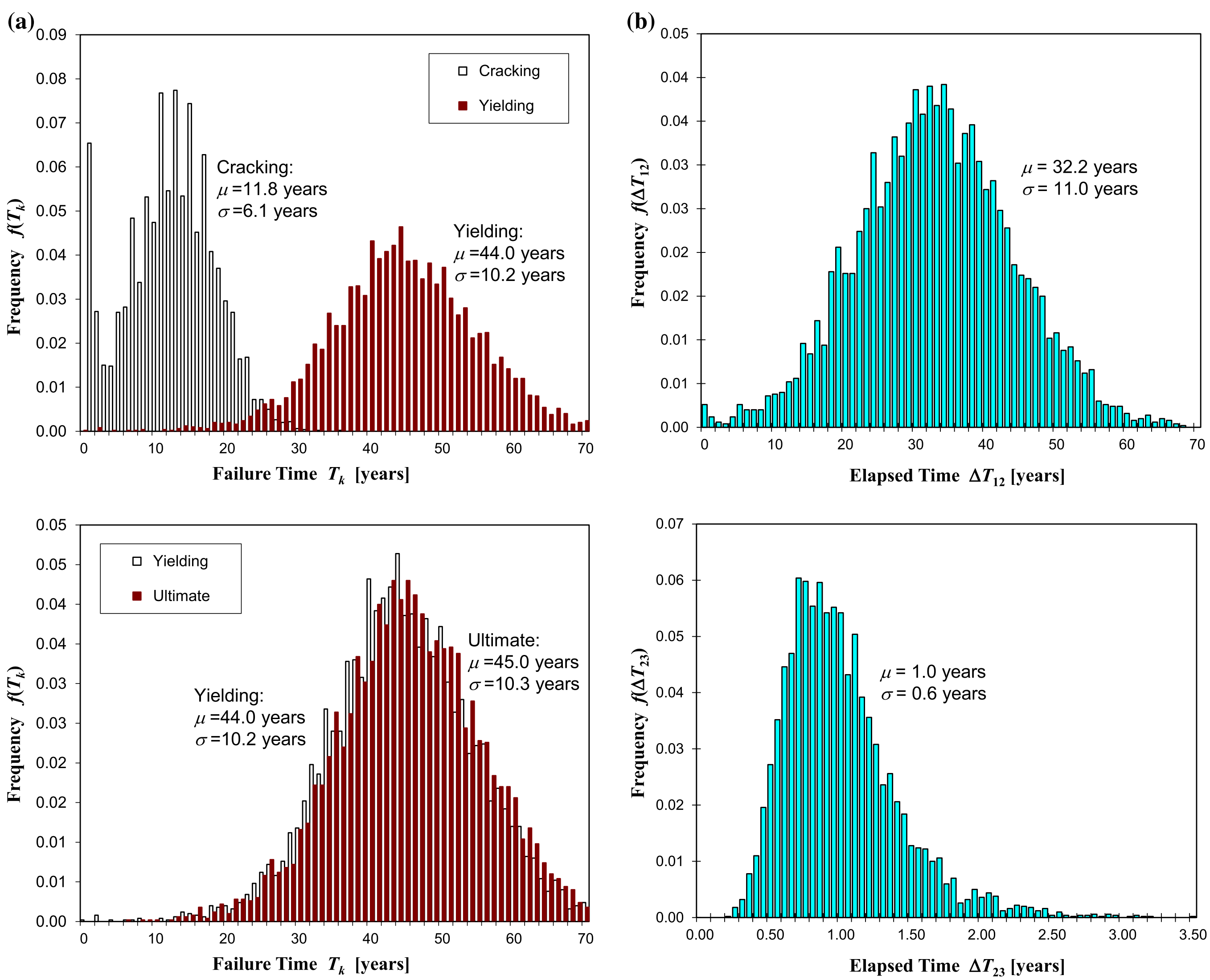

Figure 14. PMFs of (a) failure times T_1, T_2, T_3, and (b) elapsed times between failures ΔT_{12} and ΔT_{23} associated to the occurrence of the sequential limit states of (1) concrete first cracking, (2) steel first yielding and (3) cross-section ultimate capacity, for positive bending moment.

a limited reduction of bending strength over the first years of lifetime. However, after about 30 years of lifetime the severe deterioration of concrete in compression in the bottom slab causes a remarkable progressive decrease of structural performance in terms of both bending strength and curvature ductility.

At cross-sectional level, limit states of interest are the occurrence of local failures associated to cracking of concrete and yielding of steel reinforcement, which are warnings for initiation of damage propagation, as well as the attainment of the ultimate flexural capacity of the cross-section defined by the strain limits $\varepsilon_c = -\varepsilon_{cu}$ and/or $|\varepsilon_s| = \varepsilon_{su}$. The points associated to the limit states of (1) concrete first cracking, (2) steel first yielding, and (3) cross-section ultimate capacity, are indicated on the capacity curves shown in Figure 11. The corresponding time evolution of the $k = 1,2,3$, failure load multipliers $\lambda_k = \lambda_k(t)$, computed for the design values $M^+ = 4$ MNm and $M^- = -6$ MNm of the bending moments λM^+ and λM^-, is shown in Figure 12.

The reserve of load capacity after concrete cracking ensures a suitable level of overall structural redundancy for both positive and negative bending moment, as shown in Figure 13 in terms of redundancy factor $\Lambda_{13} = \Lambda_{13}(t)$ between (1) cracking and (3) ultimate capacity of the cross-section. For positive bending moment, redundancy increases continuously over time, even if the structural performance in terms of load capacity decreases. For negative bending moment, redundancy exhibits a moderate increase during the first period of exposure to damage, and rapidly decreases after about 30 years of lifetime. These results indicate that corrosion of steel reinforcement in tension, even though it involves a reduction of load capacity, may have beneficial effects in terms of structural redundancy. Contrary, the effects of deterioration of concrete in compression are generally detrimental to structural redundancy.

The reserve of load capacity after steel yielding is instead very limited and does allow for significant redundancy in between yielding and ultimate states. In this case the elapsed time between failures provides useful information about the available time to repair after a local yielding occurs.

With reference to a target load multiplier $\lambda^* = 1.0$, the failure times T_1, T_2, T_3, and the related elapsed times between failures ΔT_{12}, ΔT_{23}, associated to the occurrence of the sequential limit states of (1) concrete first cracking, (2) steel first yielding, and (3) cross-section ultimate capacity, are listed in Table 3 for the nominal system under positive and negative bending moments.

The uncertainty effects on these performance indicators are investigated also in probabilistic terms based on the probabilistic information given in Table 1. Based on 5000 Monte Carlo simulations, Figure 14 shows the PMFs of the failure times T_1, T_2, T_3 (Figure 14(a)), and elapsed time between failures ΔT_{12}, ΔT_{23} (Figure 14(b)) for the case of positive bending moment.

The deterministic and probabilistic results confirm that a remarkable rapidity of repair may be required after occurrence of a severe local failure, such as yielding of steel reinforcement. Moreover, failure loads and failure times associated to concrete cracking or other minor local failure events could provide warnings of more severe future damage states or critical threats and to support in this way the decision-making process for maintenance and repair planning.

Conclusions

Failure loads and failure times of deteriorating RC structures have been investigated. Life-cycle performance indicators, related to time-variant structural redundancy and elapsed times between sequential failures occurring under continuous deterioration processes, have been formulated. The effects of the damage process on the structural performance have been evaluated by considering uncertainties based a methodology for life-cycle assessment of concrete structures exposed to the diffusive attack from environmental aggressive agents. The proposed approach has been applied to the assessment of structural redundancy and elapsed time between failures of a RC frame and a RC bridge deck under corrosion.

The results show that the prediction of the local and global failure modes and of their occurrence in time provides useful information on the remaining life-cycle of deteriorating RC structures. In fact, after local failures occur, a very fast repair may be required under a severe exposure scenario to prevent structural collapse. Failure times and time intervals between subsequent failures must be computed for this purpose, since other damage-tolerance performance indicators, such as structural redundancy, do not provide a direct measure of the failure rate.

Therefore, failure times and elapsed time between failures are important performance indicators to be used jointly with other performance measures, such as reliability, redundancy, robustness, resilience, and sustainability for a rational approach to life-cycle design, assessment and maintenance of deteriorating structure and infrastructure systems. This approach is clearly more demanding than standard time-invariant design procedures, since it involves the modelling of complex deterioration processes and the evaluation of several performance indicators over the structural lifetime. For this reason, reliable deterioration modelling of materials and structural components and computationally efficient structural analysis procedures considering time effects, as those presented in this paper, are essential to a robust prediction of the time-variant structural performance and to support and advance the civil engineering profession in this field.

However, it is worth mentioning that deterioration models are generally very sensitive to change in the probabilistic parameters of the input random variables and their robust validation and accurate calibration are difficult tasks to be performed due to the limited availability of data. Further efforts aimed at gathering new data from both existing structures and experimental tests are crucial for a successful calibration and implementation in practice of the presented approach. Also efforts in the modelling of nonlinear structures using finite elements with time-variant properties (Biondini & Vergani, 2015; Biondini et al., 2004a), probabilistic finite element analysis (Biondini, Bontempi, Frangopol, & Malerba, 2004b; Biondini & Frangopol, 2008b; Teigen, Frangopol, Sture, & Felippa, 1991a, 1991b), reliability-based inspections (Onoufriou & Frangopol, 2002), probabilistic importance assessment of structural components (Gharaibeh, Frangopol, & Onoufriou, 2002), and developing improved models for cost and risk estimation of maintenance actions (Frangopol & Kong, 2001; Saydam & Frangopol, 2015) are necessary to ensure protection of civil infrastructure facilities over time at minimum life-cycle cost.

Disclosure statement

No potential conflict of interest was reported by the authors.

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