A Measure of Lifetime Structural Robustness

Author:

Fabio Biondini, Department of Structural Engineering, Politecnico di Milano, P.za L. da Vinci, 32, 20133 Milan, Italy, biondini@stru.polimi.it

ABSTRACT

The concepts of structural robustness, disproportioned failure and progressive collapse are usually associated to damage suddenly provoked by accidental actions and abnormal loads. However, damage could also arise gradually in time from aging of structures. Depending on the damage propagation mechanism, such kind of damage may also involve disproportionate effects. For this reason, it is of great interest to develop suitable measures of lifetime structural robustness. In this paper, the time-variant robustness of deteriorating concrete structures is investigated with respect to the ultimate limit state of collapse. The effects of the damage process are evaluated by using a proper methodology for lifetime assessment of concrete structures exposed to diffusive attacks from environmental aggressive agents. The evolution of the collapse load is evaluated by means of a time-variant limit analysis. The percentage residual carrying load capacity of the structure is related to the total amount of structural damage and, in this form, assumed as suitable lifetime measure of structural robustness. This measure is finally used to formulate a robustness criterion to verify if a structural system is robust or weak. The proposed approach is illustrated through the application to the lifetime robustness assessment of an existing reinforced concrete arch bridge.

INTRODUCTION

Structural robustness is recognized as a fundamental property of structural systems to prevent the occurrence of damage propagation phenomena and to mitigate the risk from disproportionate failure events and progressive collapse [Ellingwood 2006]. In this context, structural robustness evaluations are usually related to damage suddenly provoked by accidental actions and abnormal loads, such as explosions or impacts. In fact, the first developments in the field followed the collapse of the Ronan Point building in London in 1968 after a relatively small gas explosion. More recently, a new rise of interest on structural robustness followed other notable building collapse events caused by terrorist attacks, including the Alfred P. Murrah Federal Building in Oklahoma City in 1995, and the Twin Towers at the World Trade Center in New York in 2001.

There are, however, other sources of damage that, depending on the damage propagation mechanism, may lead to disproportionate consequences. These include damage arising gradually in time from aging of structures. For example, in concrete structures damage may be induced by diffusive attacks from environmental aggressive agents, like sulfates and chlorides. To promote a life-cycle oriented approach to design and maintenance of robust structures, it is thereby of great interest to develop suitable measures of lifetime structural robustness with respect to a progressive deterioration of the system performance.

The concept of robust structures is still an issue of controversy. In fact, despite procedures aimed to identify weak links within a structure have been reported in literature [see for example Lu *et al.* 1999], there are no well established and generally accepted criteria for a consistent definition and a quantitative measure of structural robustness [Starossek and Haberland 2008]. In general, structural robustness can be viewed as the ability of the system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself. According to this definition, a measure of structural robustness should arise by comparing the structural performance of the system in the original state, in which the structure is fully intact, and in a perturbed state, in which a prescribed damage scenario is applied.

This approach has been used in Frangopol and Curley [1987] and Frangopol *et al.* [1992] to evaluate the effects of damage on structural redundancy of truss systems formed by brittle or ductile members. Measures of time-variant system redundancy have been also recently proposed by Frangopol and Okasha [2008]. However, despite the fact that they are often used as synonymous, the terms robustness and redundancy denote different system properties [Biondini *et al.* 2008]. In fact, structural redundancy can be defined as the ability of the structural system to redistribute among its members the load which can no longer be sustained by some other damaged members.

Based on a comparison of the system performance in the intact state and in a damaged state, measures of time-variant structural robustness under prescribed damage propagation mechanisms have been proposed in Malini [2005], Restelli [2007], and Biondini and Restelli [2008]. In particular, the effectiveness of several robustness indicators has been investigated in Biondini and Restelli [2008]. They include indicators associated with the properties of the structural system only, such as eigenvalues and conditioning number of the stiffness matrix, and indicators also depending on the loading scenario, such as stored energy, displacements, and pseudo-load vectors equivalent to the effects induced by damage (forward pseudo-loads) or associated to repair interventions aimed to fully restore the system performance (backward pseudo-loads).

The general approach presented in Biondini and Restelli [2008] is followed in this paper to formulate a measure of lifetime structural robustness of concrete structures subjected to diffusive attacks from environmental aggressive agents with respect to the ultimate limit state of collapse. The effects of the damaging process are evaluated by using a proper methodology for lifetime assessment of concrete structures exposed to aggressive environments [Biondini *et al.* 2004], and are described at the cross-sectional level in terms of time evolution of axial force-bending moment resistance domains. The corresponding evolution of the collapse load is evaluated by means of a time-variant limit analysis [Biondini and Frangopol 2008].

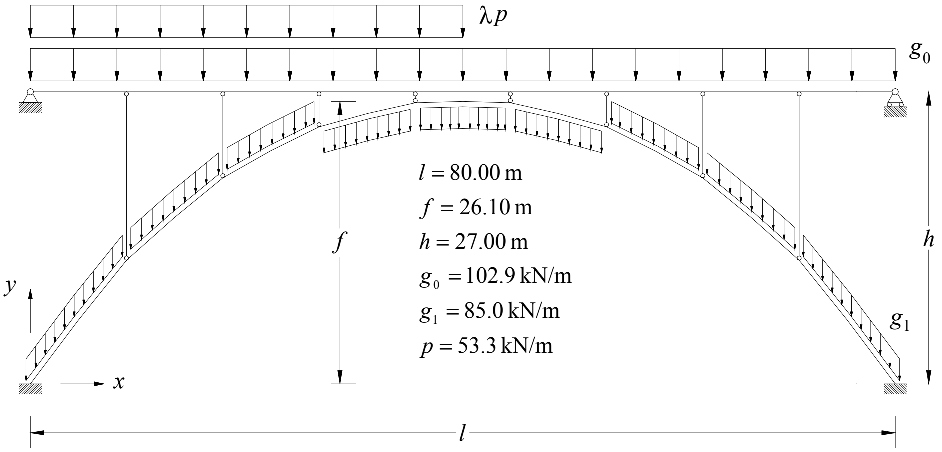
Since the methodology for lifetime assessment is based on a local description of the deterioration process, a global measure of damage is obtained at each time instant by means of a weighted average of the local damage of the materials over the volume of the structure. In this way, the percentage residual load capacity is related directly to the amount of global damage and, in this form, assumed as suitable lifetime measure of structural robustness. This measure is finally used to formulate a robustness criterion to verify if a structural system is robust or weak.

The proposed approach is illustrated through the application to the lifetime structural robustness assessment of an existing reinforced concrete arch bridge. The deterministic results presented in this paper are expected to provide a basis for a probabilistic formulation of the lifetime robustness aimed to promote a life-cycle reliability-based design and maintenance planning of robust structures.

DAMAGE EFFECTS ON THE LIFETIME STRUCTURAL PERFORMANCE

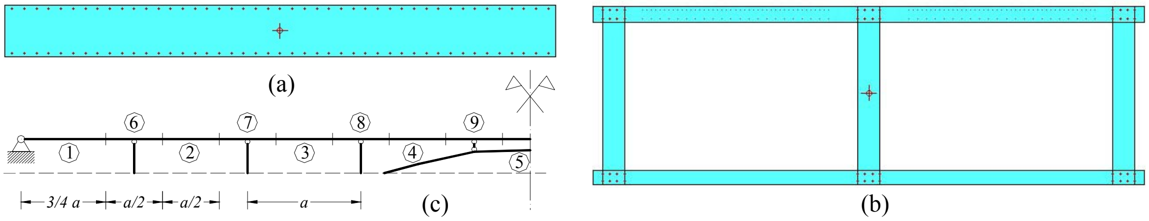
Case Studied

The present study refers to the arch bridge over the Corace river in Italy [Galli and Franciosi 1955]. The structural scheme and the overall dimensions of the bridge are shown in Figure 1. The arch has a rectangular cross-section with dimensions 0.57×6.00 m, and it is reinforced with 45+45=90 steel bars with diameter ∅28 mm (Figure 2.a). The beam has a two-cellular cross-section with main dimensions 2.00×6.00 m (Figure 2.b). The other dimensions are: web thickness = 0.20 m; top slab thickness = 0.18 m; bottom slab thickness = 0.16 m. The distribution of the reinforcement along the beam is given in Table 1 with reference to the subdivision shown in Figure 2.c. The material strengths are f_c=30 MPa for concrete in compression, and f_s=300 MPa for steel. The structure is subjected to a set of dead loads g and to a live load p, as shown in Figure 1.



Arch Shape	
x [m]	y [m]
0.00	0.00
4.44	5.90
8.89	11.43
13.35	15.39
17.80	19.04
22.20	21.53
26.70	23.76
31.20	24.99
35.60	25.98
40.00	26.10

FIGURE 1 - STRUCTURAL MODEL OF THE ARCH BRIDGE.



(a) (c) (b)

FIGURE 2 – DETAILS OF THE CROSS-SECTIONS AND REINFORCEMENT LAYOUT: (A) ARCH; (B) BEAM; (C) BEAM SUBDIVISION (SEE TABLE 1).

Span	1	2	3	4	5	6	7	8	9
A_s'	21∅28 130∅8	48∅28 130∅8	42∅28 130∅8	30∅28 130∅8	24∅28 130∅8	48∅28 130∅8	48∅28 130∅8	45∅28 130∅8	33∅28 130∅8
A_s	21∅28	30∅28	42∅28	24∅28	24∅28	21∅28	36∅28	27∅28	24∅28

TABLE 1 - REINFORCEMENT OF THE BEAM (SEE FIGURE 2.C).

Modeling of Structural Damage

The bridge is assumed to be subjected to a diffusive attack from an environmental aggressive agent located along the free edges of both beam and arch. The time-variant performance of the bridge under this damage scenario has been investigated in Biondini and Frangopol [2008] by using a general approach to the lifetime assessment of concrete structures in aggressive environments [Biondini *et al.* 2004]. In this approach the diffusion process is simulated by means of a special class of evolutionary algorithms called cellular automata, and the structural damage induced by diffusion is modeled by introducing a degradation law of the effective resistant area of both concrete matrix and steel bars. Details of the methodology can be found in Biondini *et al.* [2004]. Based on the analysis presented in Biondini and Frangopol [2008], the damage effects induced by diffusion during the first 50 years of lifetime are shown in Figures 3 and 4 in terms of resistant bending moments of the axially unloaded beam (Figures 3.a and 3.b), and axial force-bending moment resistance diagrams of the arch (Figure 4.a). The deterioration of the supporting walls is not investigated since they are assumed as not critical with respect to collapse.

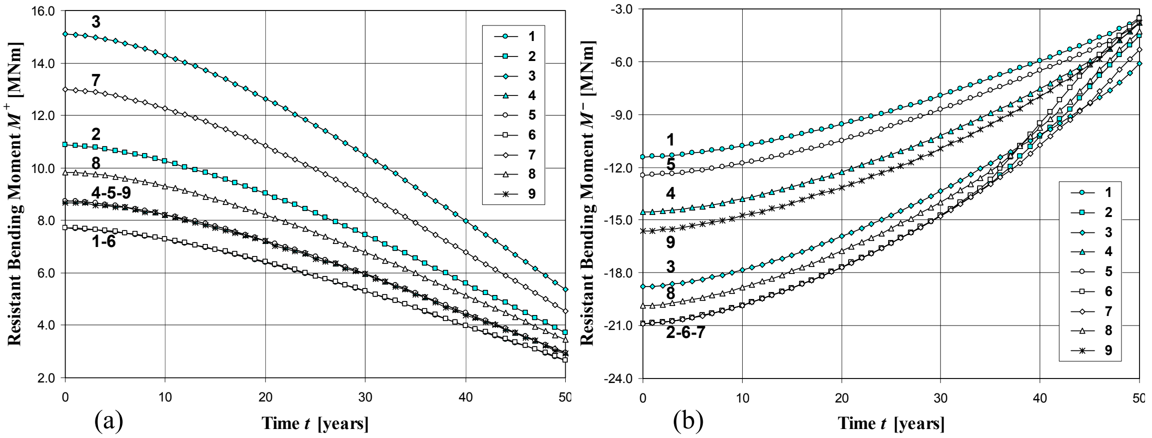


FIGURE 3 – TIME EVOLUTION OF (A) POSITIVE AND (B) NEGATIVE RESISTANT BENDING MOMENTS OF THE BEAM (SEE TABLE 1).

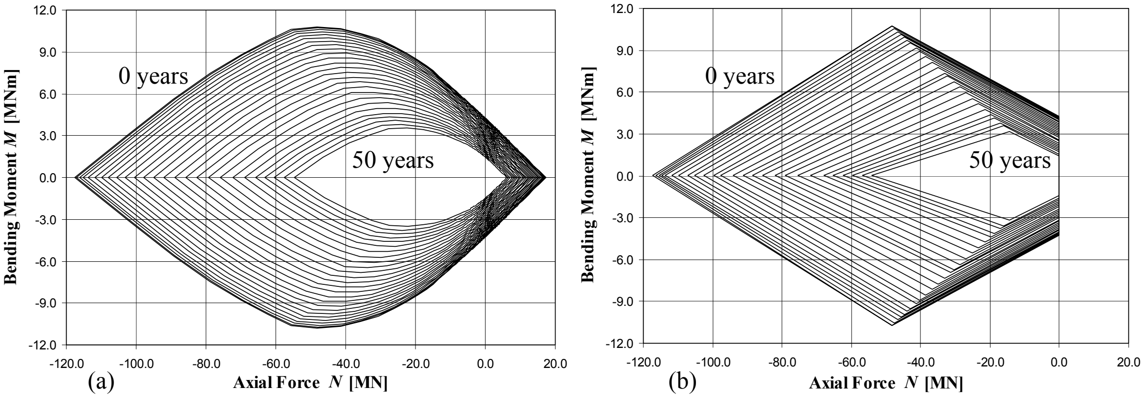


FIGURE 4 – TIME EVOLUTION OF THE AXIAL FORCE–BENDING MOMENT RESISTANCE DIAGRAMS OF THE ARCH ($\Delta t = 2$ YEARS): (A) ACTUAL DOMAINS; (B) STEPWISE LINEARIZATION.

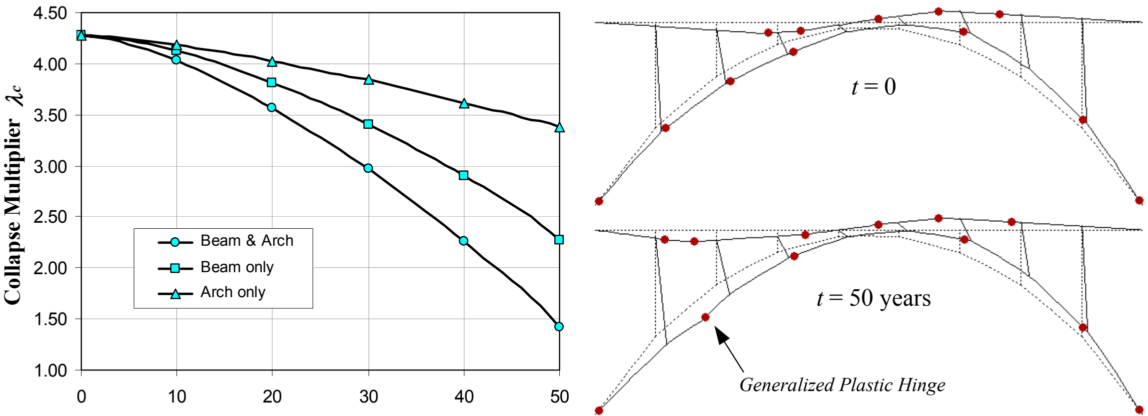
Time-variant Limit Analysis

Let $\lambda \geq 0$ be a scalar multiplier of the live loads. By assuming that the structure is safe for $\lambda = 0$, the collapse multiplier λ_c associated with structural collapse can be obtained from the theorems of limit analysis under the hypotheses of perfectly plastic behavior and negligible second order effects. In spite of such idealizations, limit analysis can be successfully applied to concrete structures by assuming a suitable effective value of the concrete compression strength (Nielsen 1999). Moreover, the limit analysis problem can be conveniently formulated as a linear programming problem if a stepwise linear approximation of the resistance domains is adopted.

For the arch bridge under investigation the carrying load capacity deteriorates over time due to the damage process. Therefore, a time-variant limit analysis is required to evaluate the collapse multiplier $\lambda_c=\lambda_c(t)$. Based on the resistant bending moments of the beam shown in Figure 3, and by assuming the four-side stepwise linearization of the axial force-bending moment resistance diagrams of the arch shown in Figure 4.b, the time evolution of the collapse multiplier is evaluated by solving the corresponding linear programming problem at several time instants.

A time-variant limit analysis under the assumption of (I) simultaneous deterioration of both beam and arch has been carried out in Biondini and Frangopol [2008]. In the present study, two scenarios with (II) damage of the beam only, and (III) damage of the arch only, are also investigated. The time evolution of the collapse multiplier $\lambda_c=\lambda_c(t)$ for the three investigated cases is shown in Figure 5.a. The comparison of the results indicates that case (I) is the worst scenario, as expected, and that the effects of damage in case (II) are higher than in case (III). In all cases the damage process leads to a significant reduction of the collapse multiplier. It is worth noting that such reduction is associated to a noteworthy redistribution of the internal stress resultants, with a consequent modification of the collapse mechanism over time, as shown in Figure 5.b for case (I).

If developed in a probabilistic context, such results allow to assess the lifetime structural reliability [Biondini and Frangopol 2008]. However, for robustness evaluations the variation of the collapse multiplier has to be directly related to the amount of damage to provide meaningful information.



(a) (b)

FIGURE 5 – (A) TIME EVOLUTION OF THE COLLAPSE MULTIPLIER λ_C FOR THREE DAMAGE SCENARIOS: (I) DAMAGE OF BOTH BEAM AND ARCH; (II) DAMAGE OF THE BEAM ONLY; (III) DAMAGE OF THE ARCH ONLY. (B) COLLAPSE MECHANISM AT THE INITIAL TIME ($\lambda_C = 4.28$) AND AFTER 50 YEARS OF LIFETIME ($\lambda_C = 1.42$) FOR SCENARIO (I).

LOCAL AND GLOBAL MEASURES OF DAMAGE

The methodology applied for the lifetime assessment of the bridge is based on a local definition of damage, which develops at the material level due to the diffusive environmental attack [Biondini *et al.* 2004]. In particular, the material damage at point **x** and time t is described by dimensionless damage indices $\delta_c=\delta_c(\mathbf{x}, t)$ and $\delta_s=\delta_s(\mathbf{x}, t)$ for steel and concrete, respectively, which provide a measure of damage within the interval [0; 1]. Such indices provide a comprehensive description of the damage evolution over the structure. However, due to their *local* nature, they do not seem handy for global evaluations of system robustness. A more synthetic *global* measure of damage may be derived from δ_c and δ_s by a weighted average over the volume of the materials [Biondini 2004]. By denoting $\Delta_c=\Delta_c(t)$ and $\Delta_s=\Delta_s(t)$ the contribution of concrete and steel, respectively, the global damage index $\Delta=\Delta(t)$ can be defined at the cross-sectional level as follows:

$$\Delta(t) = [1-\omega(t)]\Delta_c(t) + \omega(t)\Delta_s(t) \tag{1}$$

$$\Delta_c(t) = \frac{\int_{A_c} w_c(\mathbf{x},t)\delta_c(\mathbf{x},t)d\mathbf{x}}{\int_{A_c} w_c(\mathbf{x},t)d\mathbf{x}} \tag{2}$$

$$\Delta_s(t) = \frac{\sum_m w_{sm}(t)\delta_{sm}(t)A_{sm}}{\sum_m w_{sm}(t)A_{sm}} \tag{3}$$

where $\omega=\omega(t)$, $w_c=w_c(\mathbf{x},t)$, and $w_{sm}=w_{sm}(\mathbf{x},t)$ are suitable weight functions, A_c is the area of the concrete matrix, and A_{sm} is the area of the m^{th} steel bar. This cross-sectional formulation can be extended at the structural level by an average integration over all members of the system.

In case any portion of material volume is expected to play a specific role in the damage process, suggested values for the weights are $w_c=w_{sm}=1$ and $\omega=(f_s A_s)/(f_c A_c)$, where ω is the mechanical ratio of reinforcement. Figure 6.a shows the corresponding time evolution of the global damage index Δ obtained for the arch bridge for the three investigated damage scenarios.

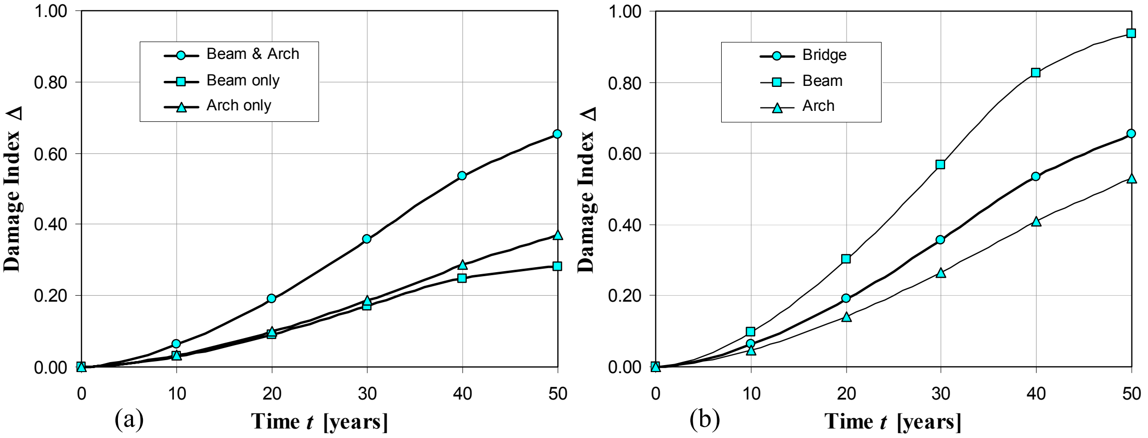


FIGURE 6 – (A) TIME EVOLUTION OF THE GLOBAL DAMAGE INDEX Δ FOR THREE DAMAGE SCENARIOS: (I) DAMAGE OF BOTH BEAM AND ARCH; (II) DAMAGE OF THE BEAM ONLY; (III) DAMAGE OF THE ARCH ONLY. (B) CASE (I) WITH THE CONTRIBUTIONS OF BEAM AND ARCH.

The comparison of the results shown in Figure 6.a confirms that case (I) is the worst scenario. However, it is interesting to note that the amount of global damage in case (II) is lower than in case (III), even if the reverse has been obtained in terms of damage effects on the collapse multiplier (Figure 5.a). This can be explained by noting that the arch, due to its larger volume, plays a more important role than the beam in the definition of the global damage, as highlighted for case (I) in Figure 6.b, where the separate contributions of beam and arch are shown.

LIFETIME STRUCTURAL ROBUSTNESS

The limit load multiplier $\lambda_c=\lambda_c(t)$ is a suitable performance indicator with respect to collapse, and its ratio to the initial value $\lambda_{c0}=\lambda_c(0)$ provides an effective time-variant measure of structural performance within the range [0, 1]:

$$\rho(t)=\frac{\lambda_c(t)}{\lambda_{c0}} \tag{4}$$

The knowledge of the time function $\rho=\rho(t)$ is in general not sufficient to formulate a measure of structural robustness. In fact, as already pointed out, the variation of the collapse multiplier has to be compared with the corresponding amount of damage to provide meaningful information for robustness evaluations. This goal can be achieved by relating the index $\rho=\rho(t)$ to the global damage $\Delta=\Delta(t)$. In this way, the functional $\rho=\rho(\Delta)$ can be regarded as a robustness index. This index can be effectively used to compare the robustness associated to different systems and damage scenarios. Figure 7.a shows the relationships $\rho=\rho(\Delta)$ obtained for the arch bridge for the three investigated damage scenarios. The comparison of such results clearly indicates that, for the same amount of damage Δ, the robustness ρ is maximum for case (III), intermediate for case (I), and minimum for case (II). Consequently, in case (II) the lifetime robustness is lower than in case (I), despite that a reverse tendency is observed for the collapse load (Figure 5.a).

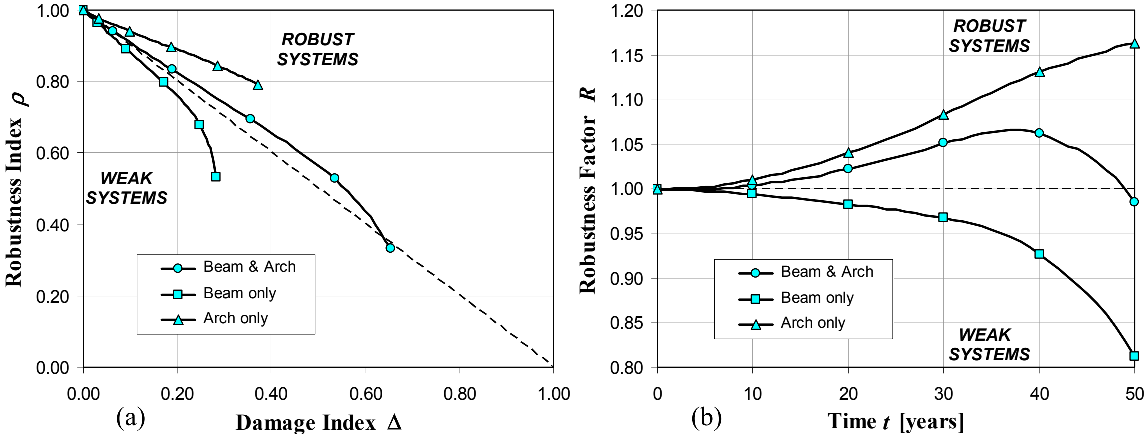


FIGURE 7 – (A) ROBUSTNESS INDEX ρ VERSUS GLOBAL DAMAGE Δ, AND (B) TIME EVOLUTION OF THE ROBUSTNESS FACTOR R WITH $\alpha=1$ FOR THREE DAMAGE SCENARIOS: (I) DAMAGE OF BOTH BEAM AND ARCH; (II) DAMAGE OF THE BEAM ONLY; (III) DAMAGE OF THE ARCH ONLY.

The robustness index $\rho=\rho(\Delta)$ can also be used to formulate a robustness criterion to verify if a structural system is robust or weak. The following criterion is proposed:

$$R(\rho,\Delta)=\rho(t)^{\alpha}+\Delta(t)^{\alpha}\geq 1 \tag{5}$$

where $R=R(\rho, \Delta)$ is a robustness factor, and α is a shape parameter of the boundary $R=R(\rho, \Delta)=1$. The structural system is robust when the criterion is satisfied ($R\geq1$), and weak otherwise ($R<1$). The value of the parameter α can be properly selected according to the acceptable level of damage susceptibility for the structure under investigation. A value $\alpha=1$, which indicates a proportionality between acceptable loss of performance and damage, should be appropriate in most cases. Moreover, values $\alpha<1$ should be avoided, since they allow for disproportionate damage effects, and values $\alpha>1$ could be required for structures of strategic importance.

Figure 7.b shows the time evolution of the robustness factor $R=R(t)$ of the arch bridge computed for $\alpha=1$ for the three investigated scenarios. The corresponding linear boundary $\rho=(1-\Delta)$ of the robustness criterion is represented by a dashed line in Figure 7.a. The diagrams shown in Figures 7.a and 7.b highlight that the bridge is robust for cases (I) and (III), and weak for case (II). It is worth noting that the time evolution of the robustness factor $R=R(t)$ may also provide useful information to plan eventual repair interventions and maintenance actions.

CONCLUSIONS

A measure of lifetime structural robustness of deteriorating systems with respect to structural collapse has been proposed. The results of the studied application demonstrated the effectiveness of the proposed measure in comparing the robustness associated to different systems and damage scenarios. Moreover, a robustness criterion aimed to judge if a structural system is robust or weak has been presented. This criterion may also be used to plan eventual repair interventions and maintenance actions to protect, improve and/or restore the lifetime system performance.

The proposed approach has been developed in a deterministic context and under the hypotheses of plastic behavior and negligible second order effects. However, it can be clearly generalized by improving the structural model to account for limited ductility and large displacements, and by considering the uncertainties involved in the problem. Future developments will focus on a probabilistic formulation of the lifetime robustness aimed to promote a life-cycle reliability-based design and maintenance planning of robust structures.

REFERENCES

[1] Biondini F. A Three-dimensional Finite Beam Element for Multiscale Damage Measure and Seismic Analysis of Concrete Structures. *13th World Conference on Earthquake Engineering*, Paper No. 2963, Vancouver, B.C., Canada, August 1-6, 2004.

[2] Biondini, F., Bontempi, F., Frangopol, D.M., Malerba, P.G., Cellular Automata Approach to Durability Analysis of Concrete Structures in Aggressive Environments, *Journal of Structural Engineering*, ASCE, **130**(11), 2004, 1724-1737.

[3] Biondini, F., Frangopol, D.M., Probabilistic Limit Analysis and Lifetime Prediction of Concrete Structures, Taylor & Francis, *Structure and Infrastructure Engineering*, **4**(5), 2008, 399-412.

[4] Biondini, F., Frangopol, D.M., Restelli, S., On Structural Robustness, Redundancy and Static Indeterminacy, *ASCE Structures Congress 2008*, Vancouver, B.C., Canada, April 24-26, 2008.

[5] Biondini, F., Restelli, S., Damage Propagation and Structural Robustness, *First International Symposium on Life-Cycle Civil Engineering* (IALCCE'08), Varenna, Lake Como, Italy, June 10-14, 2008. In *Life-Cycle Civil Engineering*, F. Biondini and D.M. Frangopol (Eds.), CRC Press, Taylor and Francis Group, 2008.

[6] Ellingwood, B.R., Mitigating Risk from Abnormal Loads and Progressive Collapse. *Journal of Performance of Constructed Facilities*, ASCE, **20**(4), 2006, 315-323.

[7] Frangopol, D.M., Curley, J.P., Effects of Damage and Redundancy on Structural Reliability, *Journal of Structural Engineering*, ASCE, **113**(7), 1987, 1533-1549.

[8] Frangopol, D.M., Iizuka, M., Yoshida, K., Redundancy Measures for Design and Evaluation of Structural Systems, *Journal of Offshore Mechanics and Arctic Engineering*, ASME, **114**(4), 1992, 285-290.

[9] Frangopol, D.M., Okasha, N.M., Probabilistic Measures for Time-variant System Redundancy, *Inaugural International Conference of the Engineering Mechanics Institute* (EM08), Minneapolis, Minnesota, USA, May 18-21, 2008.

[10] Galli, A., Franciosi, V., Il calcolo a rottura dei ponti a volta sottile ed impalcato irrigidente. *Giornale del Genio Civile*, **11**, 1955, 686-700 (In Italian).

[11] Lu, Z., Yu Y., Woodman, N.J., Blockley, D.I., A theory of structural vulnerability. *The Structural Engineer*, **77**(18), 1999, 17–24.

[12] Malini, A., *Structural Robustness of Frame Systems*, Dissertation, Politecnico di Milano, Milan, Italy, 2005 (In Italian).

[13] Nielsen, M.P., *Limit Analysis and concrete plasticity*, CRC Press, 1999.

[14] Restelli, S., *Measure of Structural Robustness of Deteriorating Systems*, Dissertation, Politecnico di Milano, Milan, Italy, 2007 (In Italian).

[15] Starossek, U, Haberland, M., Measures of structural robustness – Requirements & applications. *ASCE Structures Congress 2008*, Vancouver, B.C., Canada, April 24-26, 2008.