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Evolutionary design of structural systems with time-variant performance

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This paper presents an evolutionary approach to the optimal design of structural systems with time-variant performance. The complexity of a multidimensional design is handled by introducing a set of quality indices, which represent the multiple targets of design, and by a systemic vision of the structural model, which can be viewed either as a whole having its own emerging properties, or as composed of elementary parts with their own specific characteristics. An overall measure of lifetime structural quality, able to also take the time-variant structural performance into account, is then obtained through a weighted integration of the quality indices over the expected service life of the structure. In addition, in order to overcome the limits of mathematical optimization methods in solving design problems of high complexity and multidimensionality, the optimal structural morphology is searched for through a two-level heuristic approach based on biologically inspired evolutionary procedures. Finally, the application of the optimal design to a cable-stayed bridge is presented. The results of this evolutionary design process highlight the important role played by suitable measures of structural quality and show that the optimal configurations strongly depend on the time-variant structural performance.

Keywords: Structural damage; Lifetime performance; Structural quality; Systemic vision; Multidimensional design; Evolutionary optimization

1. Introduction

The continuous development of a growing human sensibility towards the quality of engineering structures is favouring a new vision of the conceptual design process, which nowadays must be aimed to find structural forms not only mechanically efficient, technically feasible, inexpensive and safe, but that also comply with more general satisfaction criteria, like human comfort, aesthetics, ecological needs, among others. In addition, the high costs and drawbacks usually involved in maintenance and rehabilitation interventions on existing structures have recently demonstrated that durability requirements are not yet adequately considered at the design stage. This aspect points out that a conceptual design of durable structures must account for the desired set of performance criteria not only at the initial time of construction, but also during the whole expected service life, despite the deteriorating effects induced by the unavoidable sources of mechanical damage.

Based on such considerations, this paper presents a novel approach to the evolutionary design of structural systems characterized by multiple and time-variant performance criteria, as well as by a high number of design variables. Therefore, for the sake of synthesis, in this study the design problem is defined as a “multidimensional design” to synthetically recall its expected multidimensionality with respect to both the design targets and the design variables.

In this context, the concept of sustainability in structural engineering is firstly presented and the basic targets that should be considered to design a sustainable structure are

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then identified. In order to handle the complexity inherent in a multidimensional design, these multiple targets are quantitatively translated into a set of quality indices, which allow a synthetic judgement about the goodness of a design solution and a direct comparison among different design alternatives. In particular, the definition of such indices takes advantage of a systemic vision of the structural model, which can be viewed either as a whole having its own emerging properties, or, at the same time, as composed of subsystems with their own specific characteristics (Biondini and Marchiondelli 2003).

Subsequently, the optimal design of deteriorating structural systems for a given durability is addressed by a proper measure of structural quality that accounts for the time evolution of structural damage by means of a suitable material degradation law. Starting from this time-variant indicator of structural performance, a new quality index, leading to a measure of the lifetime structural efficiency, is then obtained through a weighted integration of this function over the expected service life of the structure (Biondini and Marchiondelli 2004).

Finally, in order to overcome the limits of classical mathematical-based optimization methods in solving design problems of high complexity and multidimensionality, the optimal structural morphology is searched for through a two-level heuristic approach based on biologically inspired evolutionary procedures which operate on the basis of some analogies with the growing and evolutionary processes of natural systems (Biondini *et al*. 2002).

The effectiveness of the proposed procedure is pointed out by means of the optimal design of the structural morphology of a cable-stayed bridge. The results of the evolutionary design process highlight the important role played by suitable measures of structural quality and show that the optimal configurations strongly depend on the time-variant structural performance.

2. Sustainability and multidimensional design

2.1 *Analytical versus synthetic design*

An awareness of the many factors influencing the quality of a structure requires a radical change in the usual conception of the structural design process. In the past, the designer, supported by his experience, intuition and cultural background, usually proceeded by discarding the solutions judged unfeasible and by choosing a structural scheme coherent with the basic technical and architectural requirements. The actual performance of the tentative scheme emerged from a series of structural analyses, which the designer utilized to make the adjustments required to identify a final design.

This way of proceeding (that we call analytical design as it looks for a specific solution in the universe of design alternatives) does not give any guarantee that the adopted solution is actually the best one. In fact, in this analytical process, the designer usually refers to well-known schemes and to the results of analyses aimed to assess their performance. In this way, the designer forgets the actual role of calculus in design, born as an aid to the general understanding of the problem, and not as an alternative to the capability of making design choices and discovering new design solutions. For these reasons, analytical design tends to produce repetitive structural schemes and, consequently, to limit the artistic sensibility of the designer which, on the contrary, is by nature daring and innovative.

Lately, the designer has rediscovered his actual need: to achieve a synthesis, or to find, among all the possible configurations, the structure that best satisfies the desired requirements. In this relatively new design process (that we call synthetic design as it tends to explore the whole universe of design alternatives), the best solution is implicitly chosen through the preliminary definition of its target performances (Vincenti 1990). In this crucial phase, the designer must have a full and comprehensive vision of all the aspects influencing the quality of the final solution, not only from a structural point of view, but also with reference to other performances usually considered of secondary importance in the past, such as the durability of the structure, the correct insertion in its context, the environmental impact, the aesthetic value, among others.

This approach clearly makes the design problem very complex and then less suitable to be formalized within the classical optimization theory (Simon 1981). In fact, the typical mono-objective formulations looking for the most economical solution must be replaced with more coherent multi-objectives formulations accounting for the actual complex nature of the problem. Moreover, the widening of the quality concept of a structure imposes a multidisciplinary design strategy, where the synergetic cooperation among several different fields (e.g. technical, economical, social, ecological) is required to effectively solve a multidimensional design problem.

2.2 *Design targets*

The challenge for current designers resides in the management of the complexity involved in the existing relationships among the human society, the natural environment and the artificial built-up environment. In order to safeguard the ecosystem of the planet and the quality of human life, the new way of design must account for the global impact of any structure during its whole life cycle, on both the environment and society, not only with reference to the present generation, but with reference to the future ones too.

These needs can be synthesized with the word sustainability, which, in recent years has assumed a wider and wider meaning aimed to sensitize a change of mentality and the adoption of a life style respectful of all the resources of the planet. Based on these concepts, the principle of sustainability invites the designer to conceive a structure, not as something closed that provides shelter from the external agents, but to relate it to the surrounding environment and to make the structure contemporarily agreeable to humans and respectful of the loading capacity of the ecosystem.

A first step towards this new conception of design should clearly account for the previously mentioned multidimensionality of the problem. In particular, a basic set of targets, which should be considered as the driving criteria for the design of a sustainable structure, can be synthesized as follows:

Structural efficiency. The structure of a system must have sufficient strength and stability characteristics, with an overall stress distribution that allows a good exploitation of both the material properties and the structural volume.

Functionality. A structure must be able to comply in a satisfactory, ergonomic and comfortable way with the functions for which it has been designed. The structure should therefore also be characterized by sufficient flexibility and versatility to allow changes of use.

Preservability. This characteristic indicates the ability of a structure to protect itself from the many potential sources of damage, caused for instance by exceptional events (fires, earthquakes, hurricanes, landslides, and others) or by the ordinary interaction with aggressive environments (for instance carbonation, corrosion, freezing and unfreezing cycles).

Economy. The cost of a structure includes not only the cost of construction, but also the cost of management and maintenance during the whole service life, as well as the cost of final dismission at the end of this period. Another important aspect deals with the social cost of a structure. In fact, the production and use of materials and services determine, on both environment and society, effects having a well defined economic impact (alterations of the ecosystems, pollution, injuries to human health, loss of value of material goods, non-enjoyability of immaterial goods, etc.), with long term consequences on future generations too.

Aesthetics. This property synthesizes the exterior pleasantness of a structure, its architectonic originality and artistic value. Furthermore, aesthetics represents the most immediate method to express the social value of a building and to acquaint the society and community with it. For these reasons, it is essential to pay great attention to factors such as geometrical order and regularity, structural simplicity and clearness, lightness and harmony of the proportions, either of the structure as a whole, or of its elementary parts, especially in relation to the characteristics of the surrounding environment (Leonhardt 1980).

Durability. A structure must meet all the performance criteria mentioned above during its whole service life. It is necessary, therefore, to provide continuous monitoring in order to plan proper maintenance processes and eventual restorations of its initial characteristics, inevitably altered during time by the activation and evolution of structural and non-structural damage processes.

The three targets of structural efficiency, functionality and aesthetics, can be assimilated to the fundamental aspects already pointed out by Vitruvio as the essential characteristics of a structure: *firmitas*, *utilitas*, and *venustas*. In view of a multidimensional approach, we can imagine many of these targets previously introduced, together with many others thought to be equally important, as located at the vertices of a multidimensional solid completing the so-called *vitruvian triad* (see figure 1).

3. Systemic vision and design levels

3.1 *Holistic and sectorial levels*

It can be observed that every structural system is a unitary whole composed of many elementary parts, each having their own specificity, which have to be organized in an appropriate way to create an organic and durable system with its own emerging properties (see figure 2).

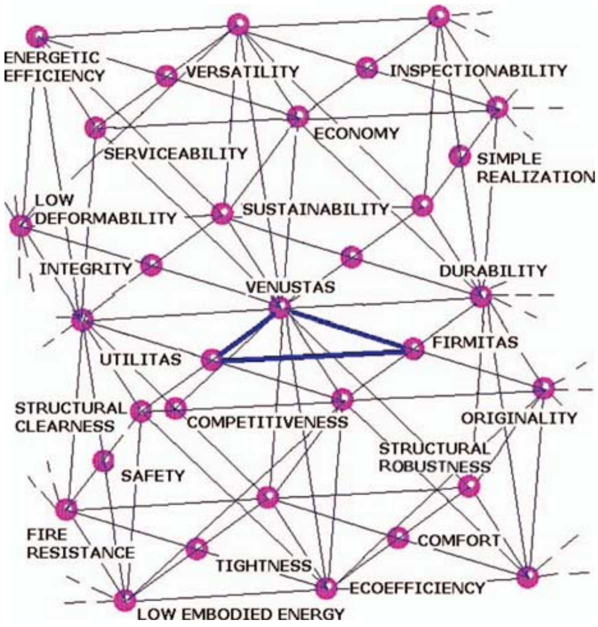

Figure 1. Targets of a multidimensional design.

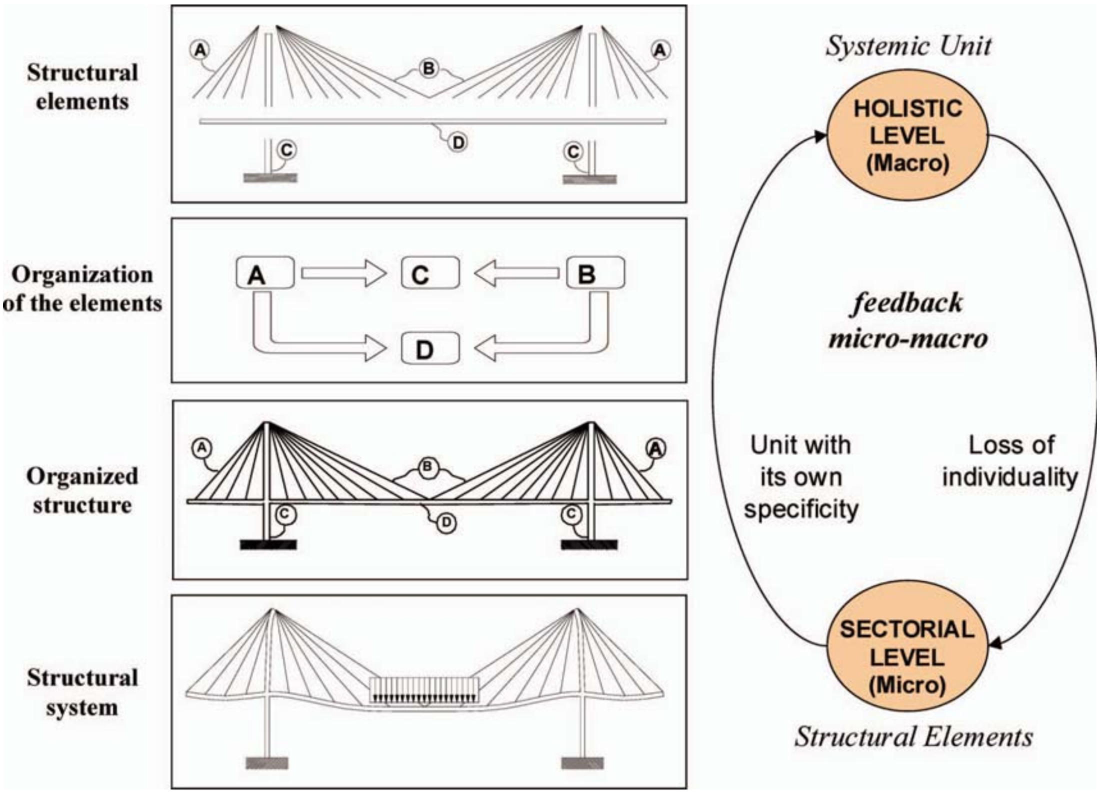

Figure 2. Systemic vision of a structure and micro-macro feedback between local and global emerging properties.

Based on this systemic vision, the design process can be set at two different levels. At the first one, which we call the holistic level, a structure is conceived as a unity, and the global quality of its emerging properties can be evaluated. At the second level, which we call the sectorial level, the subsystems forming the whole are considered separately, and the local design requirements, as well as the compatibility among the different parts of the system, can be verified. Clearly, the two levels highly interact as the status of the system derives from the status of the subsystems and, at the same time, influences it. In fact, every decision about the whole unit influences the choices regarding the substructures and, on the other hand, each single subsystem may present local demands that require some modifications of the global properties. It is important to outline that this synergetic interaction leads to feedback phenomena with global effects different to those that can be obtained from the analysis of the subsystems taken separately (von Bertalanffy 1969, Fuller 1975).

3.2 *Macro-micro optimization of the structural morphology*

According to the systemic vision of the design process, in the following, the structure is conceived as a systemic unity composed of different parts, called zones. A zone may be composed of either a portion or a whole structural member, as well as of a set of such members. Each zone, considered singly, has its own specificity (sectorial level), while it loses its individuality within the unity (holistic level). This hierarchical duality, already pointed out at the conceptual level (figure 2), is now translated at the methodological level by means of a hierarchical macro-micro optimization process of the structural morphology.

At the macro level, the structure is viewed as a systemic unity whose behaviour is obtained from the structural relationships existing among its zones. The aim of the macro optimization is the search for the optimal external morphology, i.e. the best static scheme and the best sizing, shape and topology of the global system. At the micro level, the attention is focused on the single zone and the micro optimization aims to determine the optimal internal morphology, i.e. the best sizing, topology and shape of the structural members.

4. Biologically inspired evolutionary procedures

4.1 *Heuristic versus mathematical optimization methods*

From a theoretical point of view, a multidimensional design problem can be formulated as a multi-objective problem and can be treated with classical optimization methods. Of course, in structural engineering, mathematical optimization appears to be a powerful explorative tool for a rational search strategy, especially in solving well-defined mono-objective problems. This has also recently been proved in the context of a lifetime-oriented structural optimization (Azzarello *et al.* 2006). However, mathematical methods may be unsuitable to handle the high complexity and multidimensionality of the actual design problem, which is usually not convex and characterized by multiple local optima, unreliable gradients, mixed integer/continuous design variables, among other drawbacks. For these reasons, mathematical optimization is often not effective in finding the harmonization that resolves the contrast existing among the multiple design objectives. This goal can reasonably be achieved only by means of extreme simplifications of the design model, sometimes so large that the nature of the original problem itself is lost.

A possible way to overcome this drawback is to adopt proper heuristic methods proven to be effective by experience. These methods do not search for the optimal solution, but simply proceed by looking for some improvements of the solutions already known. For this reason, they are usually less efficient than the classical mathematical techniques in finding the actual 'mathematical optimum', but the design solutions obtained can be considered almost optimal, and therefore good enough from the engineering point of view. In addition, due to their wide generality, heuristic methods can easily handle multidimensional design problems characterized by a high level of complexity. Consequently, heuristic almost optimal solutions of well-posed design problems are usually much better than the 'mathematical optima' provided by simpler and ill-posed models.

4.2 *Biomimetics and heuristic design*

One of the most promising heuristic methods that has recently been applied to the identification of optimal structural morphology deals with evolutionary procedures that operate on the basis of some analogies with the growing and evolutionary processes of natural systems. Such procedures belong to the field of biomimetics and are based on the simple concept that by slowly removing and/or reshaping regions of inefficient material, belonging to a given over-designed domain, the structure modifies its shape and topology, evolving towards an optimal configuration.

Based on these methods, the optimal structural morphology is found by a two-level evolutionary approach (Biondini *et al.* 2002). The external morphology, i.e. the geometrical dimensions and the topology of the structural type, is optimized at the first level by simulating the Biological Growth (BG) of natural structures like bones and trees. The internal morphology, regarding the sizing and shape of the cross-sections, is, instead selected at the second level by means of a classical Fully Stressed Design (FSD) criterion combined with a process of Evolutionary Structural Optimization (ESO), which removes elements at low stress levels. In this way, the BG procedure, which has the aim to optimize the shape of the structure, acts at the macro level, while the ESO strategy collaborates with FSD to search for the optimal internal morphology at the micro level. The basic steps of the overall procedure, in which the micro optimization follows the macro optimization, should be repeated until convergence occurs.

Clearly, on the basis of the desired effect, it is possible to keep only one of the two above-mentioned levels active, and to optimize only particular details or zones of the structure. However, the synergetic interaction between macro and micro optimization usually leads to better results than those obtained when the procedures work separately. Moreover, it is worth noting that in the approach presented here, the internal morphology is modified by means of discrete design variables (commercial cross-sections), while the external morphology is optimized in a continuous way (node locations). The effectiveness and versatility of the evolutionary strategies are also pointed out by this aspect.

4.3 *Biological growth (BG) procedures*

The BG procedure allows the structure to evolve by adapting itself to the applied loads according to the axiom of uniform stress, which states that, in an optimal configuration, the stress distribution tends to be fairly regular over the structure (Mattheck and Burkhardt 1990, Mattheck 1998). The structural shape and topology therefore, are gradually modified by adding material in the zones with high stress concentrations and removing it from underloaded zones. In this paper, the axiom of uniform stress is generalized to also account for other structural and non-structural performance indicators by stating that, in an optimal configuration, the structure is characterized by a uniform performance everywhere (axiom of uniform performance).

The main steps of the BG procedure are shown in figure 3 and are briefly summarised below:

Basic step. A finite element analysis is performed to obtain the stress distribution, as well as other indicators of structural performance.

Swelling step. In the original BG procedure, the evolutionary forces that drive the swelling of the structure are based only on the stress state. In the present formulation, the driving forces $DF = DF(\tau)$ of each zone are instead computed with reference to more general measures of structural performance as follows:

$$DF(\tau) = \mathrm{A} \sum_r w_r \beta_r(\tau)[a_r(\tau) - a_r^{\mathrm{ref}}(\tau)], \tag{1}$$

where A is a suitable constant, w_r is the weight of the rth component of the driving force, $\beta_r = \pm 1$ is an evolutionary index that defines the direction of such component, and a_r^{ref} is the target value of the performance measure a_r, usually assumed with the current mean value over the structure. It is worth noting that τ denotes the evolutionary time of the search process, and must be distinguished from the real time t. The driving forces DF cause a constant swelling isotropic strain distribution $\Delta\varepsilon_{\mathrm{SW}}$ in each zone. Based on the strain vector $\Delta\mathbf{e}_{\mathrm{SW}}$ associated with $\Delta\varepsilon_{\mathrm{SW}}$, the load vector $\Delta\mathbf{f}_{\mathrm{SW}}$ equivalent to swelling is derived for each finite element e, and the corresponding incremental displacement vector of the whole structure $\Delta\mathbf{u}_{\mathrm{SW}}$ is evaluated:

$$\Delta\mathbf{f}_{\mathrm{SW}}^e = \int_{V_e} \mathbf{B}^T \mathbf{D} \Delta\mathbf{e}_{\mathrm{SW}} \mathrm{d}V \Rightarrow \Delta\mathbf{u}_{\mathrm{SW}} = \mathbf{K}^{-1} \Delta\mathbf{f}_{\mathrm{SW}}, \tag{2}$$

where V_e is the volume of the finite element, $\mathbf{B}$ is its compatibility matrix, $\mathbf{D}$ is the constitutive matrix of the material, $\Delta\mathbf{f}_{\mathrm{SW}}$ is the assembled load vector, and $\mathbf{K}$ is the stiffness matrix of the structure. It is worth noting that additional geometrical design constraints can be directly accounted for by replacing the actual boundary conditions of the swelling model so that swelling displacements that violate the constraints are not allowed. This concept is shown in figure 3, where the cantilever beam is forced to maintain its initial length during the evolution.

Update step. The location $\mathbf{x}_{i,\tau} = \mathbf{x}_i(\tau)$ of the node i of the finite element model at the current generation τ is updated according to the corresponding swelling incremental translational displacements $\Delta\mathbf{u}_{\mathrm{SW},i}$ as follows:

$$\mathbf{x}_{i,\tau+1} = \mathbf{x}_{i,\tau} + \mathrm{C}\Delta\mathbf{u}_{\mathrm{SW},i}, \tag{3}$$

where C is a suitable extrapolation factor that implicitly contains the constant A. This factor may be either considered as time-independent, or varied during the evolution. In any case, its value should be chosen to assure a gradual evolution of the structural shape and, as required for convergence, progressively decreasing driving forces.

4.4 *Evolutionary structural optimization (ESO) and fully stressed design (FSD)*

As previously mentioned, the best sizing and shape of the cross-sections of the structural members can be obtained by means of an effective collaboration between the FSD criterion, which exploits the material of each structural element in an optimal way, and the ESO procedure, which removes inefficient elements. This synergetic action leads to a powerful search strategy that we call Evolutionary Fully Stressed Optimization (EFSO).

In particular, since in the optimal structure each element is subjected to its allowable stresses under at least one load condition (Gallagher and Zienkiewicz 1973), FSD chooses, from among many available commercial profiles, the cross-section of each element so that the material is stressed at its allowable level. Since the stress state varies during the service life of the structure, FSD operates on the basis of the maximum stress values that occur at selected time instants $t = t_{\mathrm{FS}}$. Clearly, in this process the allowable stresses of the materials must be properly modified to account for both local and global instability effects. In any case, when an element has a low efficiency, FSD changes its geometrical properties by choosing a smaller cross-section.

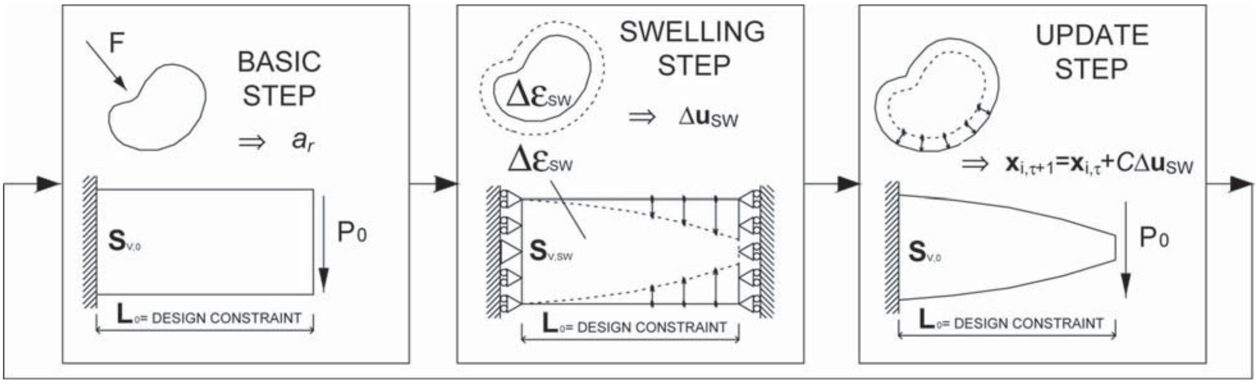

Figure 3. Fundamental steps of the BG procedure.

When the smallest available cross-section is no longer able to fully exploit the material, the ESO procedure checks the efficiency of the element and eventually removes it by degrading its constitutive properties, typically the Young modulus (Xie and Steven 1993). In the basic formulation, the minimum portion of removable material is then identified with a single finite element. However, a more general formulation can be achieved if the control of efficiency is performed on a minimum elimination unit formed by a group of elements.

5. Measure of structural quality

5.1 *Mechanical and volumetric efficiency*

The overall stress distribution in well-designed structures must allow a good exploitation of both the material properties and the structural volume. From this point of view, the structural performance depends on the stress distribution $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathbf{x})$ in each point $\mathbf{x}$ of the volume V of the structure, and it can be effectively quantified by a scalar measure of structural effort $I \geq 0$ (Musmeci 1971):

$$I = \int_V \|\boldsymbol{\sigma}(\mathbf{x})\| \mathrm{d}V, \tag{4}$$

where $\|\boldsymbol{\sigma}(\mathbf{x})\|$ is a suitable norm of the stress tensor. The structural effort can then be viewed as the volume required to realize a uniform stress state with $\|\boldsymbol{\sigma}(\mathbf{x})\| = 1$. Therefore, for a prescribed structural volume V it depends only on the structural shape.

Based on the structural effort, an index of structural efficiency $0 \leq \eta \leq 1$ can also be defined:

$$\eta = \frac{I}{\sigma_0 V}, \tag{5}$$

where $\sigma_{\max} \leq \sigma_0 \leq \sigma_{\mathrm{adm}}$ is a suitable reference stress value, σ_{adm} is the maximum admissible stress of the material, and $\sigma_{\max} = \max \|\boldsymbol{\sigma}(\mathbf{x})\|$. In this way, η represents the deviation of the global stress state from the target value σ_0. In particular, when $\sigma_0 = \sigma_{\max}$, the structural efficiency gives a measure of the good exploitation of the structural volume (volumetric efficiency η_{V}), and it is directly related to the structural topology. When the maximum admissible stress $\sigma_0 = \sigma_{\mathrm{adm}}$ is considered, the structural efficiency instead gives a measure of the good exploitation of the material properties (mechanical efficiency $\eta_{\mathrm{M}} \leq \eta_{\mathrm{V}}$), and it is directly related to structural sizing.

The previous definitions have been introduced with reference to the whole structural volume V, but they can be also applied to limited portions of the structure. In particular, for a volume subdivided into $k = 1, \ldots, n$ zones, each one with a different reference stress value σ_{0k}, the global measure of structural efficiency previously introduced can be generalized as follows:

$$\eta = \frac{\sum_{k=1}^{n} w_k \eta_k}{\sum_{k=1}^{n} w_k} = \sum_{k=1}^{n} \omega_k \eta_k = \sum_{k=1}^{n} \frac{\omega_k I_k}{\sigma_{0k} V_k} = \sum_{k=1}^{n} \frac{\omega_k \int_{V_k} \|\boldsymbol{\sigma}(\mathbf{x})\| \mathrm{d}V}{\sigma_{0k} V_k}, \tag{6}$$

where w_k is the weight coefficient of the contribution η_k associated with the zone k, and $\omega_k = w_k / \Sigma_{k=1,n} w_k$ is the corresponding unit weight. Typical choices for the weight coefficients are $w_k = 1$ (arithmetic average), or $w_k = V_k$ (volumetric average).

5.2 *Quality indices and measure of design goodness*

In order to make a hierarchical classification of the explored design alternatives possible, a direct measure of all the emerging properties that characterize the structural system is needed. A set of quality indices $a \in [0;1]$ related to the main multiple targets of a multidimensional design was introduced by Biondini *et al.* (2002) and Biondini and Marchiondelli (2003). The value $a = 1$ corresponds to the best solution, and the value $a = 0$ denotes the worst alternative. Such indices clearly depend on the design solution by means of the evolutionary time τ, or $a = a(\tau)$.

The definitions of these quality indices are given in the following. The subscript '0' refers to an initial design solution taken as reference. It is worth noting that the exponential form adopted for some indices has been proven to be effective by a series of numerical benchmarks, with values of the numerical exponents heuristically chosen in the range between 2.0 and 3.0.

Structural efficiency. This aspect depends on the stress distribution over the structure and it can be related to a good exploitation of the material properties (mechanical efficiency):

$$a_{\mathrm{ME}}(\tau) = \eta_{\mathrm{M}}(\tau) \tag{7}$$

and/or to a good exploitation of the structural volume (volumetric efficiency):

$$a_{\mathrm{VE}}(\tau) = \eta_{\mathrm{V}}(\tau). \tag{8}$$

Functionality. For serviceability purposes, this requirement can be related to the maximum displacement $s_{\max}$ over the structure:

$$a_{\mathrm{S}}(\tau) = e^{-\alpha_{\mathrm{S}} \frac{s_{\max}(\tau)}{s_{0,\max}}}. \tag{9}$$

Economy. By neglecting, for the sake of simplicity, the cost of management and maintenance interventions, the cost of the structure can be mainly associated to the material volume V:

$$a_{\mathrm{E}}(\tau) = e^{-\alpha_{\mathrm{E}}\frac{V(\tau)}{V_0}}. \tag{10}$$

Preservability. With reference to the ordinary interaction with aggressive environments, this property can be directly related to the area A of material exposed to the action of chemical agents:

$$a_{\mathrm{P}}(\tau) = e^{-\alpha_{\mathrm{P}}\frac{A(\tau)}{A_0}}. \tag{11}$$

Aesthetic. The judgement of the beauty of a structure is clearly a matter of opinion and depends on a number of subjective factors. To simplify the complexity of the problem, only the geometrical aspect is considered here, and attention is focused on properties like regularity, order, proportion and orientation of the structural members. For framed structures, composed of straight truss and beam elements, such properties can be measured as follows:

(a) Regularity and order

$$\begin{aligned} a_{\mathrm{REG}}(\tau) &= e^{-\alpha_{\mathrm{REG}}\frac{\Delta\bar{l}(\tau)}{\Delta\bar{l}_0}}, \\ a_{\mathrm{ORD}}(t) &= e^{-\alpha_{\mathrm{ORD}}\frac{\Delta\bar{\theta}(\tau)}{\Delta\bar{\theta}_0}}, \end{aligned} \tag{12}$$

(b) Proportion and orientation

$$\begin{aligned} a_{\mathrm{PROP}}(\tau) &= e^{-\alpha_{\mathrm{PROP}}\frac{\Delta\hat{l}(\tau)}{\Delta\hat{l}_0}}, \\ a_{\mathrm{ORIENT}}(\tau) &= e^{-\alpha_{\mathrm{ORIENT}}\frac{\Delta\hat{\theta}(\tau)}{\Delta\hat{\theta}_0}}, \end{aligned} \tag{13}$$

with:

$$\bar{l} = \frac{1}{m}\sum_{j=1}^{m} l_j \qquad \bar{\theta} = \frac{1}{m}\sum_{j=1}^{m} \theta_j, \tag{14}$$

$$\Delta\bar{l} = \sqrt{\frac{1}{m}\sum_{j=1}^{m}(l_j - \bar{l})^2} \qquad \Delta\bar{\theta} = \sqrt{\frac{1}{m}\sum_{j=1}^{m}(\theta_j - \bar{\theta})^2}, \tag{15}$$

$$\Delta\hat{l} = \sqrt{\frac{1}{m}\sum_{j=1}^{m}(l_j - \hat{l})^2} \qquad \Delta\hat{\theta} = \sqrt{\frac{1}{m}\sum_{j=1}^{m}(\theta_j - \hat{\theta})^2}, \tag{16}$$

where l_j and θ_j are the length and the inclination of the element $j = 1, \ldots, m$ with respect to a reference direction respectively, and $\hat{l}$ and $\hat{\theta}$ are the prescribed optimal values.

5.3 *Modelling of structural damage*

Structural damage can be viewed as a degradation of the mechanical properties that makes the structural system less able to withstand the applied actions. This damage is modelled here by introducing a degradation law of the effective resistant volume of the material:

$$\mathrm{d}V(t) = [1 - \delta(\mathbf{x}, t)]\mathrm{d}V_0, \tag{17}$$

where the subscript '0' denotes the undamaged state at the initial time $t = t_0$, and the dimensionless function $\delta = \delta(\mathbf{x},t)$ represents a damage index which provides a direct measure of the damage level within the range [0;1].

The time evolution of damage clearly depends on the physics of the deterioration process, usually related to the stress state $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathbf{x})$. Therefore, a reliable assessment of the decreasing structural performance during time requires the formulation of deterioration models suitable to describe the actual damage evolution and its interaction with the structural behaviour (Biondini *et al.* 2004). However, despite the inherent complexity of the damage process, very simple degradation models could be successfully adopted in order to define an effective hierarchical classification of the design alternatives.

5.4 *Time-variant measures of structural quality*

The deterioration of the material properties clearly affects the performance of the structural system and, as a consequence, the quality indices introduced previously are time-dependent, or $a = a(\tau,t)$. In order to define a more convenient time-invariant measure of performance, the concept of structural quality is now generalized and extended to take into account the time evolution of performance over the expected service life T of the structure. The following lifetime quality index $a_{\min}(\tau) \leq a_{\mathrm{L}}(\tau) \leq a_{\max}(\tau)$, with $a_{\min}(\tau) = \min\ a(\tau)$ and $a_{\max}(\tau) = \max\ a(\tau)$, can be introduced:

$$a_{\mathrm{L}}(\tau) = \frac{\int_0^T \psi(t)\ a(\tau, t)\ \mathrm{d}t}{\int_0^T \psi\ (t)\ \mathrm{d}t}, \tag{18}$$

where $\psi = \psi(t) \geq 0$ is a suitable weight function of the time-variant structural quality $a = a(\tau,t)$. Typical choices for such a weight function could be the following (as shown in figure 4):

(a) The lifetime quality refers only to the quality of the undamaged structure, or $a_{\mathrm{L}}(\tau) \equiv a(\tau,0)$:

$$\psi_a(0) = 1 \quad \text{and} \quad \psi_a(t) = 0\ \ \forall\ 0 < t \leq T. \tag{19}$$

(b) The lifetime quality refers only to the quality of the structure at the end of the service life, or $a_{\mathrm{L}}(\tau) \equiv a(\tau,T)$:

$$\psi_b(T) = 1 \quad \text{and} \quad \psi_b(t) = 0\ \ \forall\ 0 \leq t < T. \tag{20}$$

(c) All configurations equally contribute to the lifetime quality:

$$\psi_c(t) = 1 \ \ \forall\, 0 \le t \le T. \tag{21}$$

(d) The most efficient configurations drive the lifetime quality:

$$\psi_d(t) = a(t) \ \ \forall\, 0 \le t \le T. \tag{22}$$

(e) The least efficient configurations drive the lifetime quality:

$$\psi_e(t) = [1 - a(t)] \ \ \forall\, 0 \le t \le T. \tag{23}$$

6. Evolutionary design of a cable-stayed bridge

The evolutionary procedure presented is now applied to the optimal design of the cable-stayed bridge shown in figure 5. The loading condition is given by the self-weight of the deck and pylon, evaluated with a weight density $\gamma = 25$ kN m^{-3}, the prestressing of the stays, and a uniform load $q = 100$ kN m^{-1} acting on the deck. To avoid too high axial force in the deck during the first phase of the evolutionary process, the prestressing of the stays is initially chosen according to their inclination φ with respect to the deck axis. In particular, the initial stress value $\sigma_{p0} = 1000 \sin \varphi$ MPa is assumed. After an optimal design solution is selected, a fairly uniform prestressing level can then be achieved by modifying the

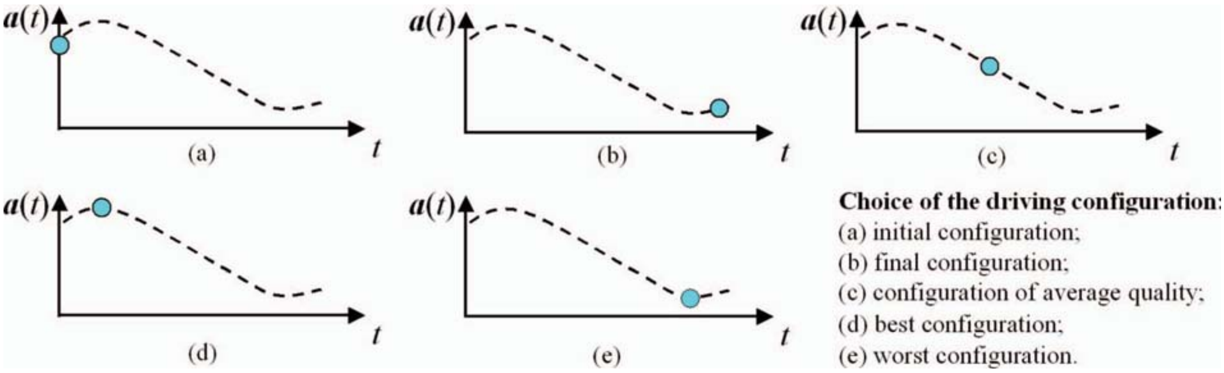

Figure 4. Some criteria for the definition of the lifetime quality index (the circle identifies the driving configuration).

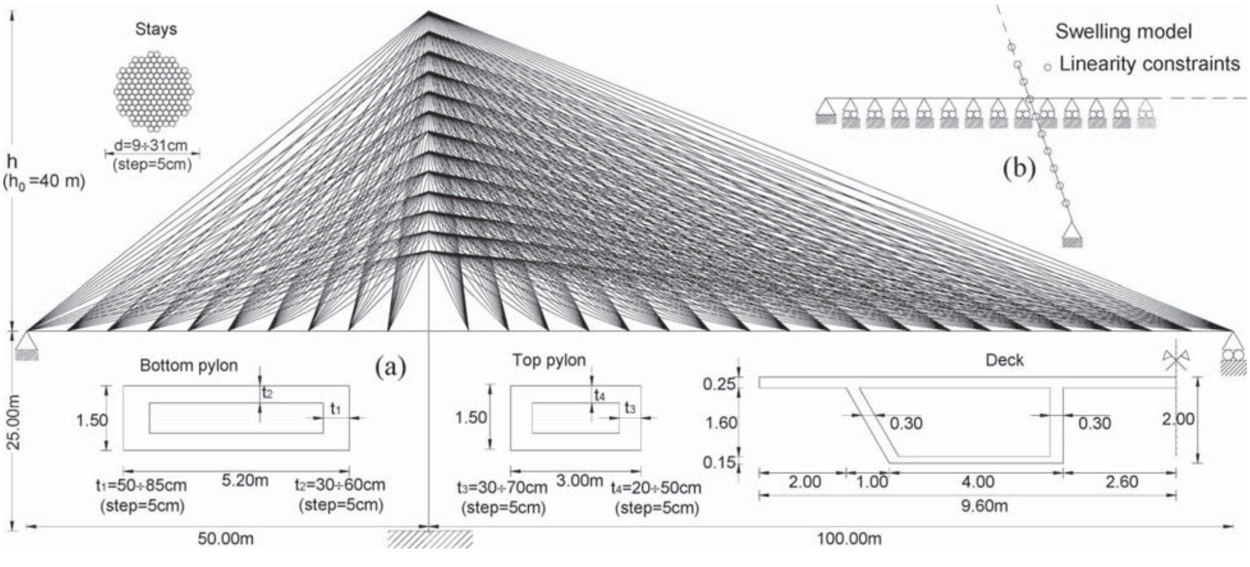

Figure 5. Cable-stayed bridge: (a) main geometrical dimensions and design variables of the initial model, and (b) swelling model and geometrical design constraints.

cross-section of the stays according to the optimal values of the prestressing forces.

To identify the optimal structural morphology, the following geometrical and topological design parameters must be defined (see figure 5(a)):

- The thickness of both the bottom and top pylon box cross-sections;
- The diameter of the stays;
- The inclination of the pylon and the height of its top part;
- The number of the stays and the location of their anchoring points along both the deck and pylon.

The initial design domain is subdivided into the following 29 zones:

- The deck (1 zone);
- The bottom and top parts of the pylon (2 zones);
- The groups of stays located on the same side with respect to the pylon and having the same anchoring point along the pylon itself (26 zones).

The swelling model, shown in figure 5(b), allows movement of the anchoring points of the stays and enforces the deck and the pylon to maintain their straight profile.

The EFSO procedure operates on the structural members on the basis of their mechanical efficiency. In particular, when the value of the efficiency is less than a given lower limit, the corresponding element is removed.

In a first application, damage is neglected and the evolutionary design is carried out with reference to the initial configurations only. Figure 6 shows some steps of the evolutionary process where the BG procedure is driven by the index of mechanical efficiency, while figure 7 makes a comparison among the solutions obtained by computing the driving forces using the other quality indices previously defined. The detailed results of these search processes are listed in table 1. The analysis of the results highlights how the evolutionary control of different quality indices can effectively drive the multidimensional design process towards structural morphologies that have the desired performance.

The evolutionary process associated with the index of mechanical efficiency is now repeated by introducing a source of damage and by accounting for the corresponding time-variant performance of the structure. A design value of the service life $T = 50$ years is chosen, and a very simple degradation model is assumed. In this model, the damage index $\delta = \delta(\mathbf{x},t) \equiv \delta(t)$ is constant over the structure and varies during time with a constant rate defined as:

$$\frac{\mathrm{d}\delta(t)}{\mathrm{d}t} = \frac{1}{T^*}, \quad \delta(t) = \frac{t}{T^*} \tag{24}$$

where T^* represents the time interval required to reach a complete damage of the material at time instant $t = T^*$, or $\delta(T^*) = 1$, starting from the undamaged state at time $t = 0$, or $\delta(0) = 0$. The damage model is defined by assuming

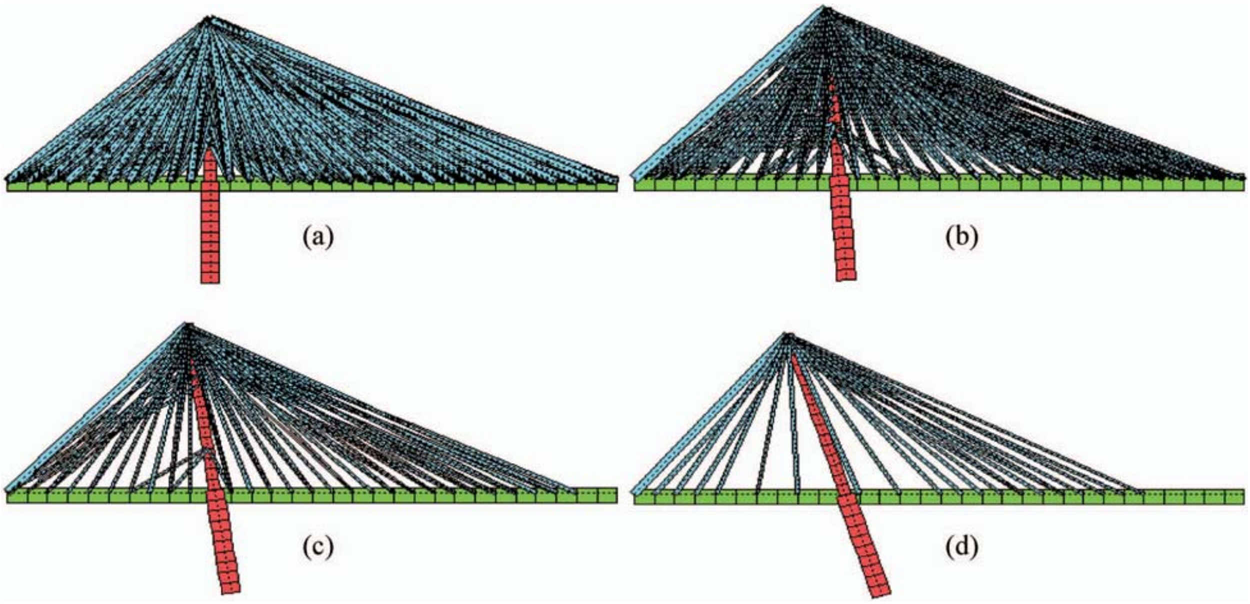

Figure 6. Some steps of the evolutionary process driven by the index of mechanical efficiency: (a) initial structure, (b) and (c) intermediate configurations, and (d) final solution.

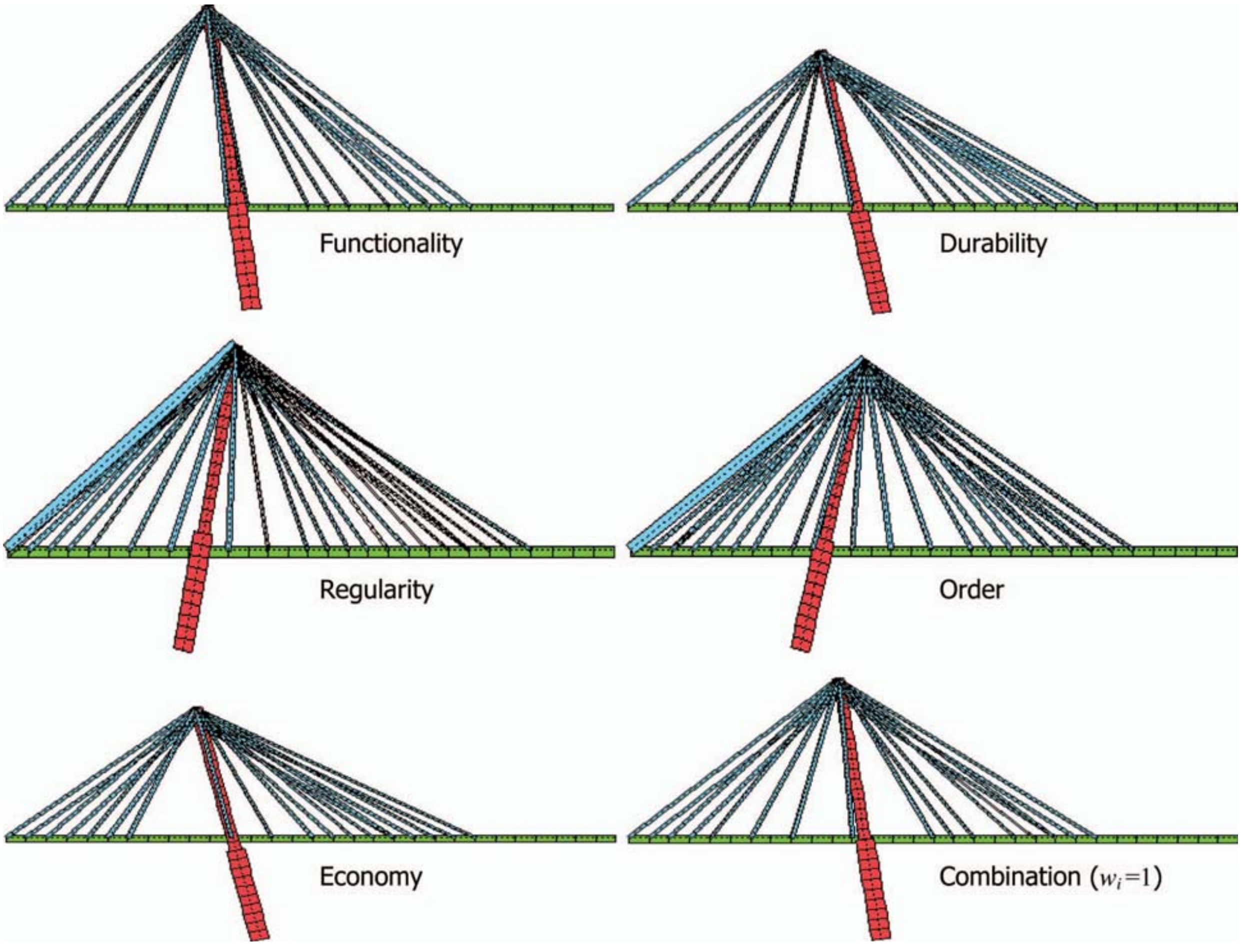

Figure 7. Some optimal time-independent configurations obtained by using different driving indices.

Table 1. Minimum and maximum values of the design variables for the optimal time-independent solutions.

Driving criterion (quality index)	t_1 (cm)	t_2 (cm)	t_3 (cm)	t_4 (cm)	d (cm)	h (m)
Mechanical efficiency	75–80	50–60	65–70	40–45	13–21	35.63
Functionality	80–80	60–60	70–75	50–55	17–25	48.62
Durability	80–85	60–65	65–70	40–45	19–27	36.32
Regularity	75–75	50–55	70–75	50–55	13–29	49.36
Order	75–75	50–55	70–75	50–55	9–31	47.08
Economy	75–80	55–60	65–70	40–45	13–27	32.48
Weighted combination ($w_i = 1$)	80–80	60–60	65–70	40–45	15–25	39.06

$T^* = 100$ years for the pier and the deck, and $T^* = 200$ years for the stays. The lifetime quality is evaluated on the basis of the time-variant mechanical efficiency, which is computed by means of linear structural analyses performed at discrete time instants during the service life of the structure, with a monitoring time interval $\Delta t = 2$ years.

Figure 8 makes a comparison among the optimal solutions associated with several evolutionary scenarios. The columns refer to different time instants t_{FS} of application of the FSD, while the rows are associated with the five (a) to (e) weight functions $\psi = \psi(t)$ previously introduced. In particular, the design solutions in column (1) have been obtained after about 300 evolutionary steps by applying the FSD at the initial time ($t_{\mathrm{FS}} = 0$), while the solutions in column (2) have been reached after about 500 evolutionary steps by operating the FSD at the end of the

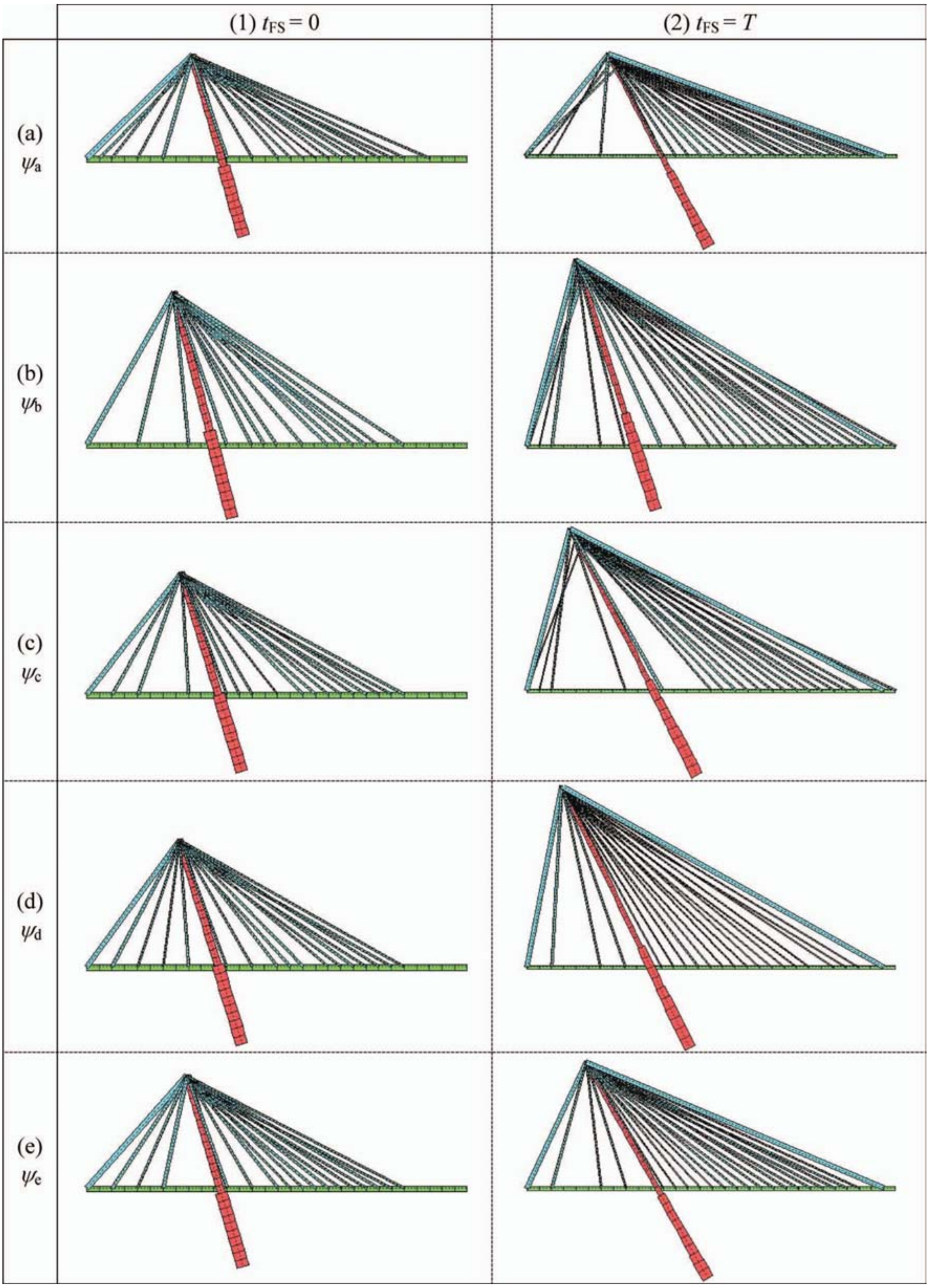

Figure 8. Optimal time-dependent solutions associated with several evolutionary scenarios obtained by operating the FSD at different time instants t_{FS} and by choosing different weight functions $\psi = \psi(t)$ for the index of mechanical efficiency a_{ME}: $\psi_a(0) = 1$ and $\psi_a(t) = 0$; $\psi_b(t) = 0$ and $\psi_b(T) = 1$; $\psi_c(t) = 1$; $\psi_d(t) = a_{ME}(t)$; $\psi_e(t) = [1 - a_{ME}(t)]$.

service life ($t_{FS} = T$). The direct comparison among the structural configurations in figure 8 shows that:

- The slope of the pylon does not depend on the adopted weight function ψ, but the pylon always rotates in such a way that its axis tends to lie along the line of action of the resultant forces exerted by the active stays.
- The choice of the weight function ψ affects the total height of the pylon and the proportions of the structural elements.
- The structural configurations (c),(d) and (e) appear to be intermediate between the limit solutions (a) and (b), which refer to the structural efficiency of the undamaged structure and of the structure at the end of the service life respectively.
- The time instant t_{FS} of application of the FSD plays a fundamental role in the definition of the optimal configurations.
- In general, the optimal configurations strongly depend on the time-variant performance of the structural system.

7. Conclusions

A novel approach to the optimal design of deteriorating structural systems has been presented. This approach is able to comply with the desired performance, not only at the initial time of construction, but also during the whole expected service life, despite the deteriorating effects induced by the unavoidable sources of mechanical damage. The main developments proposed in this paper can be summarized in the following points:

Multidimensional design. The complexity inherent in the design process is taken into consideration by searching for feasible engineering solutions complying with several targets (e.g. structural efficiency, functionality, preservability, economy, aesthetics, durability).

Quality indices. The multiple targets are quantitatively translated into a set of indices that allow a measure of the goodness of a design solution and a direct comparison among different design alternatives (hierarchical classification).

Lifetime structural quality. The proposed measures of structural quality are also extended to account for the time-variant performance by means of a weighted integration of the quality indices over the expected service life of the structure (lifetime-oriented design).

Systemic vision. In the design model, the structure is viewed either as a whole having its own emerging properties, or as composed of elementary parts with their own specific characteristics (feedback).

Heuristic methods. The search process is based on design analogies that easily handle multidimensional and complex design problems, leading to heuristic almost optimal solutions usually much better than the ‘mathematical optimums’ provided by simpler and ill-posed models (biomimetics).

The effectiveness of the proposed approach has been shown through the application to the optimal evolutionary design of a cable-stayed bridge. The analysis of the results shows how the evolutionary control of proper quality indices can effectively drive the multidimensional design process towards structural morphologies having the desired quality, and that the optimal configurations strongly depend on the time-variant structural performance.

Future developments will consider the integration of the proposed evolutionary design procedure with deterioration models suitable to describe the actual damage evolution and its interaction with the structural behaviour. Moreover, special attention should also be paid to the definition of additional weight functions for the multiple time-variant targets of a multidimensional design, e.g. economy and aesthetics, which are not directly related to structural efficiency.

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References

Azzarello, L., Biondini, F. and Marchiondelli, A., Optimal design of deteriorating structural systems, in *Proceedings of 3rd International Conference on Bridge Maintenance and Safety Management (IABMAS06)*, Porto, July 16–19, 2006.

Biondini, F., Boiocchi, S., Malerba, P.G. and Marchiondelli, A., Biologically inspired evolutionary procedures for conceptual design of the structural morphology, in *Proceedings of 2nd International Conference on Advances in Structural Engineering and Mechanics (ASEM02)*, Pusan, Korea, August 21–23, 2002.

Biondini, F., Bontempi, F., Frangopol, D.M. and Malerba, P.G., Cellular automata approach to durability analysis of concrete structures in aggressive environments. *ASCE J. Struct. Eng.*, 2004, **130**(11), 1724–1737.

Biondini, F. and Marchiondelli, A., Multidimensional design and systemic vision in structural engineering, in *Proceedings of 2nd International Conference on Structural Engineering and Construction (ISEC02)*, Rome, Italy, September 23–26, 2003.

Biondini, F. and Marchiondelli, A., Evolutionary design of durable structures, in *Proceedings of 2nd International Conference on Bridge Maintenance, Safety and Management*, (IABMAS’04), Kyoto, Japan, October 18–22, 2004.

Fuller, R.B., *Synergetics. Explorations in the Geometry of Thinking*, 1975 (Macmillan: New York).

Gallagher, R.H. and Zienkiewicz, O.C. (Eds.), *Optimum Structural Design*, 1973 (John Wiley & Sons).

Leonhardt, F., Aesthetics in structural engineering, in *Proceedings of 11th IABSE Congress*, Vienna, August 31 – September 5, 1980.

Mattheck, C., *Design in Nature. Learning from Trees*, 1998 (Springer Verlag).

Mattheck, C. and Burkhardt, S., A new method of structural shape optimization based on biological growth. *Int. J. Fatigue*, 1990, **12**(3), 185 – 190.

Musmeci, S., *La Statica e le Strutture*. Cremonese, 1971 Rome (in Italian).

Simon, H., *The Sciences of the Artificial*, 1981 (MIT Press).

Vincenti, W.G., *What Engineers Know and How They Know It*, 1990 (John Hopkins University Press).

von Bertalanffy, L., *General System Theory*, 1969 (George Braziller: New York).

Xie, Y.M. and Steven, G.P., A simple evolutionary procedure for structural optimization. *Comput. Struct.*, 1993, **49**(5), 885 – 896.