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Life-cycle multi-objective optimization of deteriorating structures

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ABSTRACT: This paper presents a new approach to life-cycle multi-objective optimization of deteriorating structures. The time-variant performance is evaluated with reference to a proper modeling of structural damage. The effects of maintenance interventions are taken into account by relating the cost of maintenance to the actual damage level. A genetic algorithm is used to solve the multi-objective optimization problem. The effectiveness of the proposed approach is demonstrated with a simple benchmark and a final application to the life-cycle optimization of a prestressed box-girder bridge. The results highlight the fundamental role of the time-variant performance and of the maintenance planning in the selection of the Pareto set of optimum design solutions.

1 INTRODUCTION

In the classical approach to multi-objective optimum structural design, the objective functions and the design constraints are defined by considering the structural performance at the initial time of construction, when the system is undamaged. However, for structures exposed to damaging environments the structural performance must be considered as time-dependent, mainly because of the progressive deterioration of the materials. For this reason, a consistent approach to optimum design of durable structures should be able to comply with the desired performance not only at the initial time of construction, but also during the overall structural lifetime, by taking into account the effects of the unavoidable sources of mechanical damage and of eventual maintenance and rehabilitation interventions.

To overcome the inconsistencies associated with a damage-free formulation of the optimum design problem, a new approach to the minimum cost design of structural systems with time-variant performance has been proposed in previous works (Azzarello *et al.* 2006, 2007). In the proposed approach, the structural damage is modeled by a proper material degradation law and the structural analysis is carried out at different time instants in order to assess the time evolution of the system performance. The design constraints are related to both the time-variant stress and displacement state, as well as to the amount of structural damage. The objective cost function is formulated by accounting for both the initial cost of the structure and the costs of possible maintenance interventions, that are properly discounted over time and assumed to be proportional to the actual level of structural damage. This procedure has been developed for truss and framed systems made by homogeneous members (Azzarello *et al.* 2006), as well as for reinforced concrete structures (Azzarello *et al.* 2007).

In this paper, the proposed approach is extended to consider multiple design targets related not only to the life-cycle cost, but also to other life-cycle performance indicators. To this aim, the design problem is formulated in the context of the multi-objective structural optimization and solved by using a genetic algorithm. A simple benchmark related to the life-cycle optimization of a tensioned bar undergoing damage is presented to demonstrate the effectiveness of the proposed approach. The multi-objective optimization procedure is finally applied to the life-cycle design of a prestressed box-girder bridge. The results highlight the important role played by both the time-variant performance and the maintenance plan in the selection of the Pareto set of optimum design solutions.

2 MODELING OF STRUCTURAL DAMAGE

With reference to truss and framed systems, damage is considered to affect the cross-sectional area $A = A(t)$, the elastic modulus $E = E(t)$, and the material strength $\bar{\sigma} = \bar{\sigma}(t)$ of each structural member:

$$A(t) = [1 - \delta_A(t)]A_0 \tag{1}$$

$$E(t) = [1 - \delta_E(t)]E_0 \tag{2}$$

$$\bar{\sigma}(t) = [1 - \delta_{\bar{\sigma}}(t)]\bar{\sigma}_0 \tag{3}$$

where $\delta_A, \delta_E, \delta_{\bar{\sigma}}$, are dimensionless damage indices which provide a direct measure of the damage level within the range [0; 1]. Proper correlation laws can be introduced to define the corresponding variation of other geometrical properties of the member cross-section, like the inertia moment.

The time evolution of the damage indices δ_A, δ_E, $\delta_{\bar{\sigma}}$, depends on the physics of the deterioration process, and it is usually related to the acting stress $\sigma = \sigma(t)$ (Figure 1a). Therefore, a reliable assessment of the time-variant structural performance requires deterioration models suitable to describe the damage evolution and its interaction with the structural behavior (Biondini *et al.* 2004). However, despite the inherent complexity of the damage laws, very simple degradation models could be successfully adopted to define an effective hierarchical classification of the design alternatives (Biondini and Marchiondelli 2006).

Without any loss of generality, in this study it is assumed that all material properties undergo the same damage process, or $\delta_A = \delta_E = \delta_{\bar{\sigma}} \equiv \delta$. Moreover, the damage index $\delta = \delta(t)$ is correlated to the stress $\sigma = \sigma(t)$ by assuming the following relationship (Figure 1b):

$$\frac{d\delta(t)}{dt} = \frac{1}{T_\delta}\left[\frac{\sigma(t)}{\bar{\sigma}_0}\right]^\alpha \qquad \bar{\sigma}_0 = \begin{cases} \bar{\sigma}_0^+ & \text{if } \sigma \geq 0 \\ \bar{\sigma}_0^- & \text{if } \sigma < 0 \end{cases} \tag{4}$$

where $\alpha \geq 0$ is a suitable constant, $\bar{\sigma}_0^-$ and $\bar{\sigma}_0^+$ are the minimum and maximum allowable stress at the initial time $t = t_0$, respectively, and T_δ represents the time period required for a complete damage under a constant stress level $\sigma(t) = \bar{\sigma}_0$ (Figure 1c). The initial condition $\delta(t_{cr}) = 0$ with $t_{cr} = \max\{t|\,\sigma(t) \leq \sigma_{cr}\}$ is assumed, where $\sigma_{cr} \leq \bar{\sigma}_0$ is a critical stress threshold.

The index $\delta = \delta(t)$ fully describes the damage evolution in each point of the structure. However, due to its *local* nature, it does not seem handy for design purposes. A more synthetic *global* measure of damage may be computed as the weighted average of δ over the volume V of the structure (Biondini 2004). For a structural system composed by p members, a global damage index can be defined as follows:

$$\hat{\delta}(t) = \sum_{i=1}^{p} \hat{\delta}_i(t) = \sum_{i=1}^{p} \int_{V_i} w_i \delta_i(t) dV \tag{5}$$

where w_i are weight functions with $\sum_{i=1}^{p} \int_{V_i} w_i dV = 1$.

3 LIFE-CYCLE COST OPTIMALITY

3.1 *Life-cycle structural cost*

The total cost C of the structure over its life-cycle is given by the sum of the initial cost C_0 and the maintenance cost C_m:

$$C = C_0 + C_m \tag{6}$$

The initial cost C_0 is computed as follows:

$$C_0 = cV_0 \tag{7}$$

where V_0 *is* the total volume of the material and c is the corresponding unit cost. For reinforced or prestressed concrete structures the cost of both materials, concrete and steel, are considered:

$$C_0 = c_c V_{c,0} + c_s V_{s,0} = c_c(V_{c,0} + cV_{s,0}) = c_c V_{c,0}^* \tag{8}$$

where $V_{0,c}$, $V_{0,s}$, are the total volumes of concrete and steel, respectively, c_c, c_s, are the corresponding unit costs, $c = c_s/c_c$ is the unit cost ratio, and $V_{c,0}^*$ is the equivalent volume of concrete.

The maintenance cost C_m can be evaluated by summing the costs of the individual interventions:

$$C_m = \sum_{k=1}^{r} \frac{C_m^k}{(1+v)^{(t_k - t_0)}} \tag{9}$$

where the cost C_m^k of each intervention $k = 1, \ldots, r$ is referred to the initial time t_0 by taking a discount rate v into account (Kong and Frangopol 2003).

3.2 *Maintenance scenario*

The previous general formulation is now specialized to a prescribed maintenance scenario. In this scenario an essential maintenance aimed to totally restore the initial structural performance is performed after each design period T_D, or at each time instant $t_k = (t_0 + kT_D)$. In this way, all the interventions have the same cost $C_m^k = C_m^1$, and the number of interventions applied during a prescribed service lifetime T_S is $r = \text{int}(T_S/T_D)$ if mod $(T_S/T_D) > 0$, or $r = [\text{int}(T_S/T_D) - 1]$ if mod $(T_S/T_D) = 0$. Therefore,

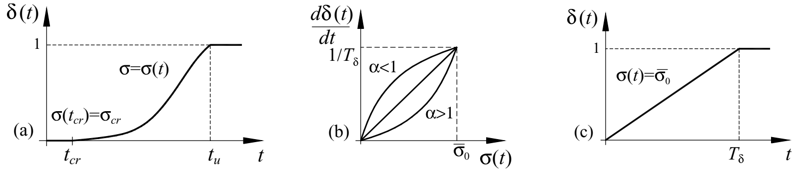

Figure 1. Modeling of structural damage. (a) Time evolution of the damage index $\delta = \delta(t)$. (b) Linear relationship between the rate of damage and the stress level $\sigma = \sigma(t)$. (c) Meaning of the damage parameter T_δ.

the cost of maintenance is:

$$C_m = C_m^1 \sum_{k=1}^{r} \frac{1}{(1+v)^{kT_D}} = C_m^1 q \quad (10)$$

where the factor $q = q(T_S, T_D, v) \leq r$ depends on the prescribed parameters T_S, T_D, and v only.

The cost C_m^1 of the single intervention is related to the level of damage developed during each design period T_D. Since damage is not recovered over time, a measure of this damage is given by the global damage index evaluated at the end of each period T_D, or:

$$\tilde{\delta} = \hat{\delta}(t_k) = \hat{\delta}(t_{k-1} + T_D), \qquad k = 1, \ldots, r \quad (11)$$

Based on the life-cycle global damage index $\tilde{\delta}$, the following linear relationship is assumed:

$$C_m^1 = C_0(\beta + \tilde{\delta}) \quad (12)$$

where β is a coefficient which takes the fixed cost of each intervention into account. In this way, the total life-cycle cost C is finally formulated as follows:

$$C = C_0[1 + (\beta + \tilde{\delta})q] \quad (13)$$

3.3 *The role of maintenance cost*

To highlight the actual role played by a prescribed maintenance program, the design of the tensioned bar shown at the top of Figure 2 is investigated. By denoting with d_0 the diameter of the undamaged cross-section and by assuming $\beta = 0$, the total cost of the bar over the prescribed service lifetime T_S is:

$$C = C_0(1 + \tilde{\delta}q) = cA_0L(1 + \tilde{\delta}q) = c\frac{\pi d_0^2 L}{4}(1 + \tilde{\delta}q) \quad (14)$$

with $\tilde{\delta} = \tilde{\delta}(d_0)$. The diameter d_0 must be chosen in such a way that the acting stress $\sigma = \sigma(t)$ is no larger than the allowable stress $\bar{\sigma} = \bar{\sigma}(t)$ over the prescribed design period T_D:

$$\sigma(t) = \frac{F}{A(t)} = \frac{4F}{\pi d_0^2[1 - \delta(t)]} \leq \bar{\sigma}(t) = \bar{\sigma}_0[1 - \delta(t)] \quad (15)$$

with $\delta(t) = \delta(d_0, t)$. Without damage ($\delta = \tilde{\delta} = 0$), the minimum cost solution d_0^* is given by the minimum diameter d_0 which satisfies the stress constraint:

$$d_0^* = d_{0,\min} = \sqrt{\frac{4F}{\pi \bar{\sigma}_0}} \quad (16)$$

On the contrary, when damage is included, the minimum cost solution d_0^* is, in general, no longer associated with the diameter $d_{0,\min}$. In fact, higher d_0^*-values may be required to achieve a balance between the maintenance cost and the amount of damage.

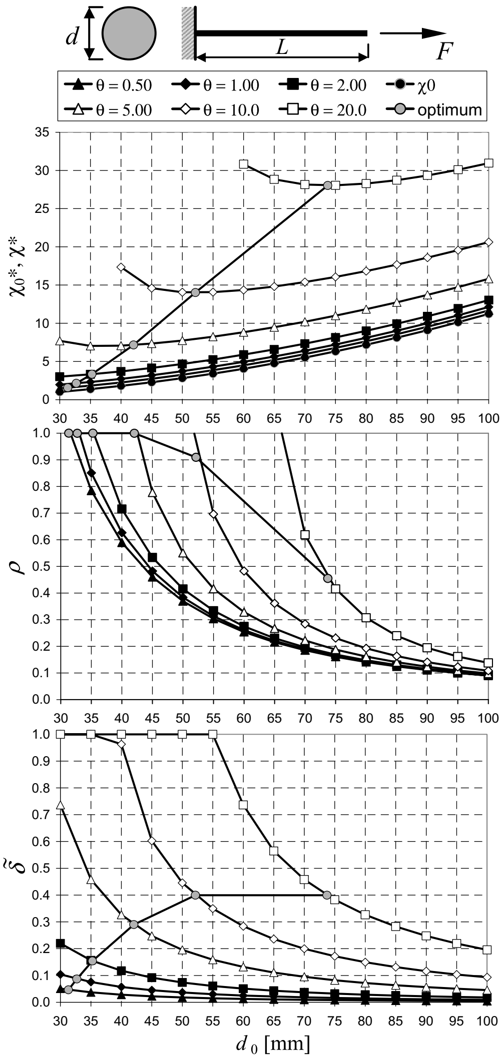

Figure 2. Initial cost χ_0^*, life-cycle cost χ^*, stress ratio ρ, and global damage $\tilde{\delta}$ for a tensioned bar undergoing damage.

These aspects are highlighted in Figure 2, where both the cost and structural efficiency of the bar versus its diameter d_0 are shown for different damage rates $\theta = T_S/T_\delta$, with $t_0 = 0$, $\sigma_{cr} = 0$, $\alpha = 1$, and with reference to the following case study: $F = 70\,\text{kN}$, $\bar{\sigma}_0 = 100\,\text{MPa}$, $T_S = 100$ years, $T_D = 10$ years, and $v = 0$ ($q = r$). The diagrams in Figure 2 refer to the following quantities:

$$\chi^* = \chi_0^* + \chi_m^* \quad \chi_0^* = C_0/C^* \quad \chi_m^* = C_m/C^* \quad (17)$$

$$\rho = \sigma(T_D)/\bar{\sigma}(T_D) \quad \tilde{\delta} = \hat{\delta}(T_D) \tag{18}$$

where $C^* = C(d_{0,\min}) = \text{cFL}/\bar{\sigma}_0$ denotes the optimal cost for $\theta = 0$. The following remarks can be made:

- The minimum feasible diameter without damage is $d_{0,\min} = 29.9$ mm. Its value increases with θ.
- The initial cost χ_0^* increases and the maintenance cost χ_m^* decreases when d_0 increases. For a given d_0, the maintenance cost χ_m^* increases with θ.
- The total cost χ^* has a minimum for $d_0^* \geq d_{0,\min}$, and the optimal diameter d_0^* increases with θ.
- The stress ratio ρ decreases with d_0 and increases with θ. The optimal solution d_0^* at the end of the design period T_D may be not fully stressed ($\rho^* \leq 1$).
- The damage index $\tilde{\delta}$ decreases with d_0 and increases with θ. For the optimal solution, such quantity tends to saturate for the higher θ-values.

Similar results are obtained by varying T_D.

4 LIFE-CYCLE STRUCTURAL OPTIMIZATION

4.1 *Multi-objective optimization problem*

The purpose of a life-cycle multi-objective design process is to find a vector of design variables $\mathbf{x} = [\ \mathbf{x}_1 \ \ \mathbf{x}_2 \ldots]^T$ which optimizes the value of a set of objective functions $\mathbf{f}(\mathbf{x}) = [\ f_1(\mathbf{x}) \quad f_2(\mathbf{x}) \ldots]^T$, according to both side constraints with bounds $\mathbf{x}^-$ and $\mathbf{x}^+$, and inequality time-variant behavioral constraints $\mathbf{g}(\mathbf{x}, t) \leq 0$:

$$\min_{\mathbf{x} \in D} \mathbf{f}(\mathbf{x}) \quad D = \left\{ \mathbf{x} | \, \mathbf{x}^- \leq \mathbf{x} \leq \mathbf{x}^+, \mathbf{g}(\mathbf{x}, t) \leq \mathbf{0} \right\} \tag{19}$$

The time-variant behavioral constraints are related to the structural response in terms of stress $\sigma_{i,\ell} = \sigma_{i,\ell}(t)$ and displacement $u_{j,\ell} = u_{j,\ell}(t)$ in each element is nodal point j, and loading condition ℓ, as follows:

$$\begin{cases} \bar{\sigma}_{i,\ell}^-(\mathbf{x}, t) \leq \sigma_{i,\ell}(\mathbf{x}, t) \leq \bar{\sigma}_{i,\ell}^+(\mathbf{x}, t) \\ \bar{u}_{j,\ell}^- \leq u_{j,\ell}(\mathbf{x}, t) \leq \bar{u}_{j,\ell}^+ \end{cases} \tag{20}$$

where $\bar{\sigma}_{i,\ell}^- = \bar{\sigma}_{i,\ell}^-(t)$ and $\bar{\sigma}_{i,\ell}^+ = \bar{\sigma}_{i,\ell}^+(t)$ are allowable stress values, $\bar{u}_{j,\ell}^-$ and $\bar{u}_{j,\ell}^+$ are prescribed displacement bounds. Constraints on structural stability, as well as on both local and global damage, can also be considered (Azzarello *et al.* 2006). In this study, the Pareto set of optimum design solutions is found by using a genetic algorithm properly formulated to effectively solve multi-objective optimization problems (Deb 2001, Biondini and Riboldi 2006).

4.2 *Life-cycle optimization of a tensioned bar*

The life-cycle design of the tensioned bar shown in Figure 2 is carried out by considering two objective functions to be minimized: the life-cycle cost C and the maximum displacement $u_{\max} = \max u(t)$. The displacement $u(t) = FL/[EA(t)]$ is computed by assuming $F = 70$ kN, $E = 210$ GPa, and $L = 5$ m. The design variables are the diameter d_0 of the undamaged cross-section and the design period $T_D \leq T_S$, with $T_S = 100$ years. A time-variant stress constraint with material strength $\bar{\sigma}_0 = 100$ MPa is considered. The life-cycle optimization problem is solved by assuming $\theta = 20$, $\beta = 0.01$, and $\nu = 0$. Figure 3 makes a comparison between the target solution of the optimization problem and the numerical results obtained by using a genetic algorithm. The following remarks can be made:

- A good agreement between the target solution and the numerical results is obtained for the Pareto set, the optimal diameter d_0, and the stress ratio ρ. For the optimal design period T_D and the global damage $\tilde{\delta}$ a deviation from the target solution is observed. This is due to the discretization adopted for d_0.
- The optimal values of the design variables d_0 and T_D increase as the life-cycle cost C increases.
- The optimal values of the design period T_D are properly selected in such a way that the ratio T_S/T_D assumes only integer values, or mod $(T_S/T_D) = 0$.
- The minimum cost solution is fully stressed and it is characterized by an optimal design period $T_D = 1$ year. However, the other Pareto optimum solutions are not fully stressed and the stress ratio ρ significantly decreases as the life-cycle cost C increases.
- The global damage $\tilde{\delta}$ tends to decrease as the life-cycle cost C increases, with abrupt variations when the optimal design period T_D changes.

Finally, Figure 4 shows the Pareto curves obtained for different values of the damage rate θ, fixed maintenance cost β, and discount rate ν. It is noted that the life-cycle cost C increases when the damage rate θ and the fixed maintenance cost β increase. This tendency is reduced by the discount rate ν.

4.3 *Life-cycle optimization of a prestressed bridge*

The proposed formulation is applied to the life-cycle optimization of the prestressed box girder bridge shown in Figure 5. The following data are assumed: $L = 40$ m, $b = 13$ m, $b_0 = 7$ m, $b_{\text{web}} = 0.75(h_{\text{bot}} + h_{\text{top}})$. The bridge is subjected to a set of applied loads: a dead load $g = \gamma A_c + 25$ kN/m, where $\gamma = 25$ kN/m^3 is the weight density and A_c is the area of the box cross-section; the live loads $q_1 = q_2 = 75$ kN/m; the prestressing loads $P = \sigma_{p0} A_s$ and $p = P\kappa$, where P is the prestressing force, $\sigma_{p0} = 1000$ MPa is the initial prestressing, A_s is the area of prestressing steel, and κ is the geometric curvature of the prestressing cable. The cable eccentricity

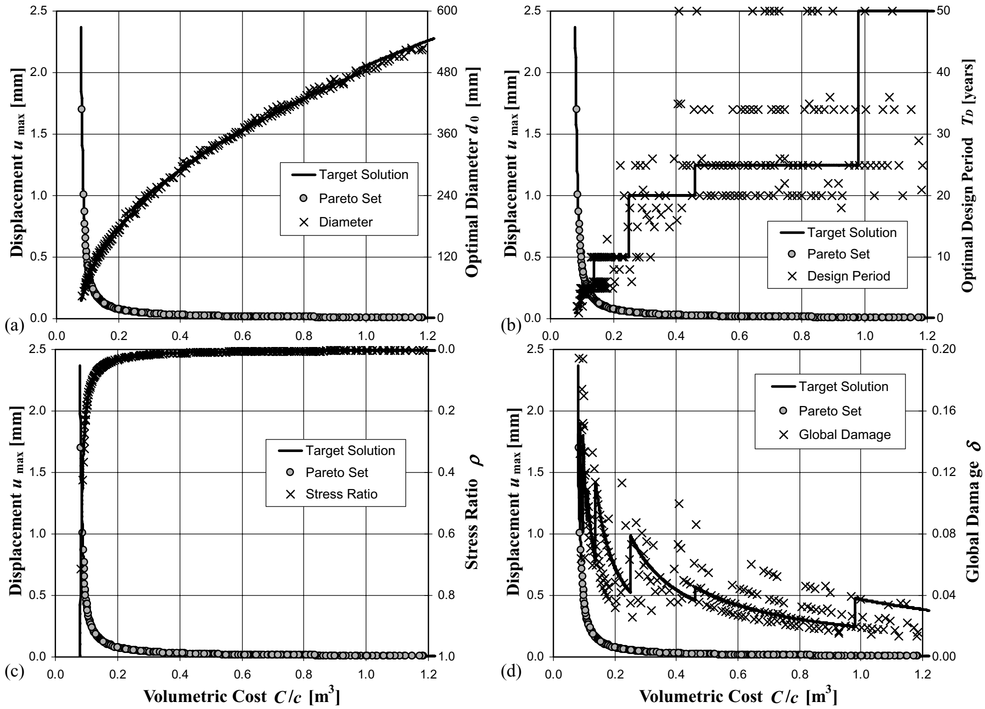

Figure 3. Tensioned bar undergoing damage. Pareto set of the optimum design solutions (life-cycle cost C versus maximum displacement $u_{\max}$) compared with (a) optimal diameter d_0, (b) optimal design period T_D, (c) stress ratio ρ, and (d) life-cycle global damage $\tilde{\delta}$ (damage rate $\theta = 20$, fixed cost of maintenance interventions $\beta = 0.01$, discount rate $\nu = 0$).

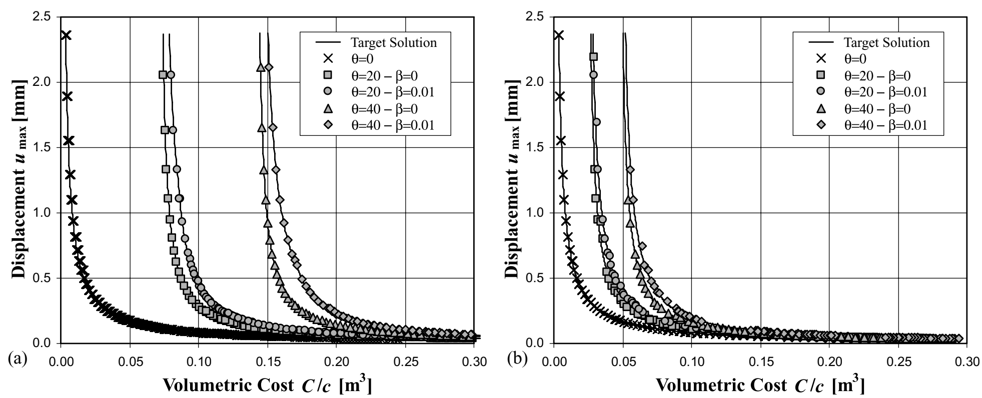

Figure 4. Tensioned bar undergoing damage. Pareto sets of the optimum design solutions (life-cycle cost C versus maximum displacement $u_{\max}$) for different values of the damage rate θ and of the fixed cost of maintenance interventions β: (a) $\nu = 0$, and (b) $\nu = 0.03$.

e is parabolic, with $e = 0$ at the ends, $e = e_1$ at the middle of each span, and $e = e_2 = 0.95\,\mathrm{h} - h_G$ at the middle support. Three load conditions are considered for the live loads: (1) $q_1 = q_2 = 0$; (2) $q_1 = q_2 = 75$ kN/m; (3) $q_1 = 0$ and $q_2 = 75$ kN/m. The damage and maintenance scenarios are defined by assuming $\theta = 0.5$, $\sigma_{cr} = 0$, $\alpha = 1$, $T_S = 100$ years, $T_D = 5$ years, $\beta = 0.05$, and $\nu = 0$ ($q = r$). The bridge is subdivided in 16 elements and the time evolution of the damage index δ is computed with reference

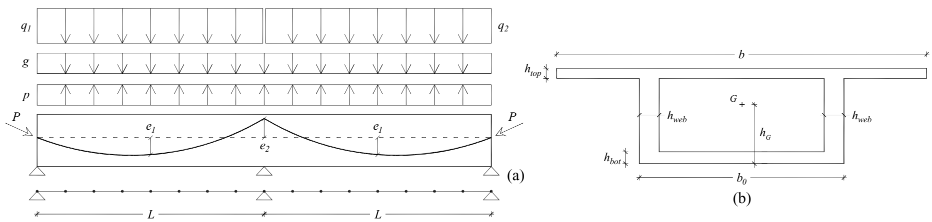

Figure 5. Box girder bridge undergoing damage. (a) Structural scheme and loading conditions. (b) Geometry of the box cross-section.

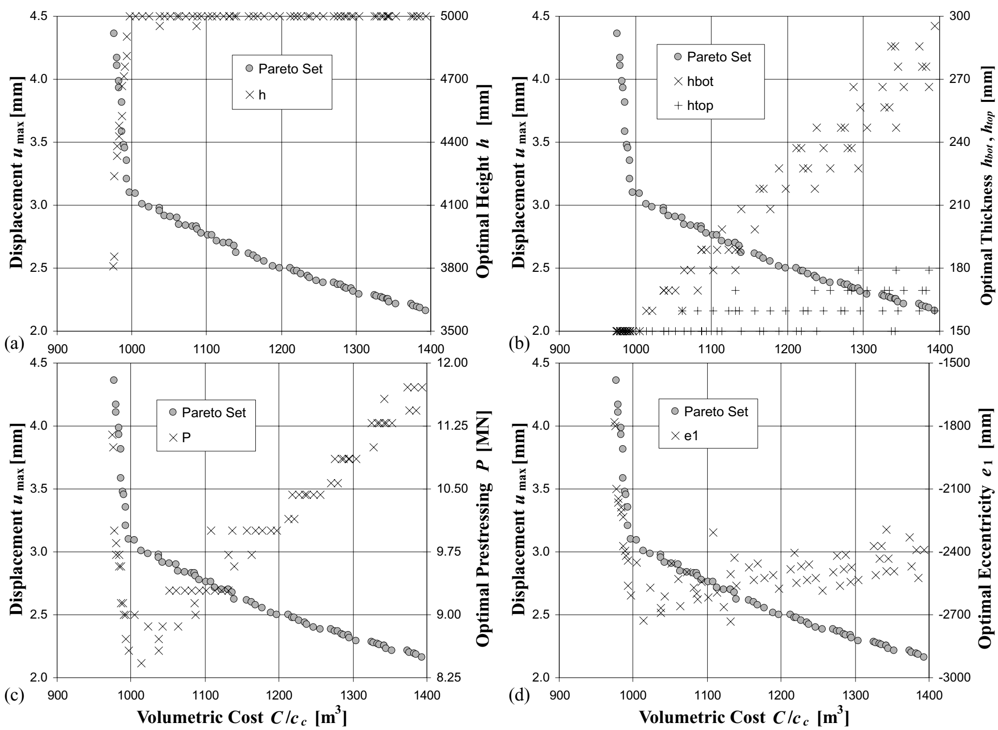

Figure 6. Box girder bridge undergoing damage. Pareto set of the optimum design solutions (life-cycle cost C versus maximum deflection $u_{\max}$) compared with the optimal values of the design variables: (a) deck height h, (b) slab thickness h_{bot} and h_{top}, (c) prestressing force P, and (d) cable eccentricity e_1 at middle span (damage rate $\theta = 0.5$, fixed cost of maintenance interventions $\beta = 0.05$, discount rate $\nu = 0$).

to the maximum stress in each segment. Based on a prescribed damage pattern, the inertia moment of the damaged cross-section is computed with a damage index $\delta' = \delta(2 - \delta)$.

The objective functions to be minimized are the life-cycle cost C and the maximum deflection $u_{\max}$ at middle span over the service life T_S. The life-cycle cost C is computed by assuming a unit cost ratio of concrete and steel $c = c_s/c_c = 100$. The displacements are evaluated with $E = 34$ GPa.

The following design variables and side constraints are assumed: height of the box section $2\text{ m} \leq \text{h} \leq 5\text{ m}$; thickness of top and bottom slabs $0.15\text{ m} \leq \text{h}_{top} \leq 0.45\text{ m}$ and $0.15\text{ m} \leq \text{h}_{bot} \leq 0.45\text{ m}$, respectively; prestressing force $7\text{ MN} \leq P \leq 16\text{ MN}$; eccentricity $0 \leq \text{h}_G + e_1 \leq h$.

The time-variant stress constraints are applied, for each loading condition, in each segment of the box girder with reference to the initial allowable stresses $\bar{\sigma}_0^- = -10$ MPa and $\bar{\sigma}_0^+ = 0$. Moreover, the displacement constraint $u_{1max} \geq 0$ on the maximum deflection at middle span under the load condition (1) is applied. Finally, the constraints $0.5 \leq h_{top}/h_{bot} \leq 2$, $A_s/A_c \leq 0.05$, and 50 mm $\leq (h_G + e) \leq (h - 50$ mm) are considered.

Figure 6 shows the results of the optimization process. It can be noted that the upper side constraint on the height h subdivides the Pareto set into two parts. This transition strongly influences the relationship between the life-cycle cost C and the variables P and e_1.

5 CONCLUSIONS

A new approach to the life-cycle multi-objective optimization of deteriorating structures under multiple loading conditions has been presented. This approach allowed to overcome the inconsistencies associated with a damage-free formulation of the optimum design problem. In fact, in the proposed formulation, the structural damage is accounted for by means of a material degradation law of the mechanical properties, and the design constraints of the optimization problem are related to the corresponding time-variant performance over the expected structural lifetime. Moreover, the life-cycle cost is evaluated by considering both the initial cost of the structure and the costs of possible maintenance interventions, that are properly discounted over time and assumed to be proportional to the actual level of structural damage.

The proposed approach has been applied to a simple benchmark related to a tensioned bar undergoing damage, and to the life-cycle optimization of a prestressed box-girder bridge under multiple loading conditions. The results proved the effectiveness of the proposed approach by highlighting that the Pareto set of optimum design solutions strongly depends on both the time-variant structural performance and the maintenance planning.

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