Life-Cycle Civil Engineering – Biondini & Frangopol (eds)
© 2008 Taylor & Francis Group, London, ISBN 978-0-415-46857-2

Damage propagation and structural robustness

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ABSTRACT: The role of damage propagation on the structural robustness of deteriorating systems is investigated. The deterioration effects are evaluated with reference to suitable indicators of the structural response. The variation of these performance indicators with respect to the values associated with the undamaged system is used to formulate dimensionless measures of structural robustness. The damage paths associated with two alternative propagation mechanisms are identified by a fault-tree analysis and described at the system level by branched networks. The effectiveness of the proposed approach is shown through applications.

1 INTRODUCTION

In structural design the concept of robust structures, or damage-tolerant structures, is often an issue of controversy, since there are no well established and generally accepted criteria for a consistent definition and a quantitative measure of structural robustness. Moreover, robustness evaluations are usually related to damage suddenly provoked by accidental actions, like explosion or impacts. However, damage could also arise slowly in time from aging of structures, as induced for example by environmental aggressive agents. In this context, it is therefore of great interest to develop suitable life-cycle measures of structural robustness with respect to a progressive deterioration of the structural performance.

In this paper, a general approach to robustness assessment of structural systems undergoing damage is presented. The deterioration effects on the system performance are evaluated with reference to suitable performance indicators identified with parameters of the structural response. The variation of these indicators with respect to the values associated with the performance of the undamaged system is used to formulate dimensionless measures of structural robustness. Moreover, an index of structural integrity aimed to quantify the severity of the structural failure with regards to its consequences, is proposed.

In the proposed approach, damage is viewed as a progressive deterioration of the material properties and its amount is specified at the member level by means of a damage index associated with prescribed patterns of cross-sectional deterioration. Starting from this local definition of damage, a model of damage propagation at the system level is also developed by considering two alternative propagation mechanisms, and by using a damage-sensitive fault-tree analysis. In such a way, all the feasible damage paths associated with the propagation mechanism and the actual topology of the system are described by branched networks where the level of activation of each nodal connection is properly tuned to account for the prescribed amount of local structural damage.

The effectiveness of the proposed approach is shown through the application to the robustness assessment of truss and frame systems.

2 STRUCTURAL PERFORMANCE INDICATORS

Strength and ductility, as well as other performance indicators of the ultimate conditions under nonlinear behavior, may result of great significance in robustness evaluations associated with damage induced by severe loadings, like explosions or impacts (Frangopol and Curley 1987, Biondini *et al.* 2008). However, performance indicators of the serviceability conditions under linear behavior, like elastic stiffness and first yielding, may become of major importance in life-cycle robustness evaluations associated with aging of structures. In the following, several performance indicators under linear elastic behavior are investigated (Restelli 2007).

2.1 *Parameters of the structural behavior*

The following performance indicators are considered:

$$d = \det(\mathbf{K}) \qquad t = \sum\nolimits_i \lambda_i(\mathbf{K}) \tag{1}$$

$$c = \frac{\max\limits_i \lambda_i(\mathbf{K})}{\min\limits_i \lambda_i(\mathbf{K})} \qquad T = 2\pi\sqrt{\max_i \lambda_i(\mathbf{K}^{-1}\mathbf{M})} \tag{2}$$

where d, t, and c are, respectively, the determinant, the trace, and the conditioning number of the stiffness matrix $\mathbf{K}$, T is the first vibration period associated with the mass matrix $\mathbf{M}$, and $\lambda_i(\mathbf{K})$ denotes the ith eigenvalue of the matrix $\mathbf{K}$. These indicators are quite general, since they are related to the properties of the structural system only. However, a structural system may have different performance under different loads. For this reason, the following indicators are also considered:

$$s = \|\mathbf{s}\| = \left\|\mathbf{K}^{-1}\mathbf{f}\right\| \quad \Phi = \frac{1}{2}\mathbf{s}^T\mathbf{K}\mathbf{s} = \frac{1}{2}\mathbf{s}^T\mathbf{f} \tag{3}$$

where $\mathbf{s}$ is the displacement vector, $\mathbf{f}$ is the applied load vector, Φ is the stored energy, and $\|\cdot\|$ denotes the euclidean scalar norm. These indicators depend on both the system properties and the loading condition, and they may refer either to the system in the original state, in which the structure is fully intact, or to the system in a perturbed state, in which a prescribed damage scenario is applied.

2.2 *Pseudo-loads*

For robustness evaluations it can also be of interest to define indicators able to simultaneously account for the structural performance of the intact system and of the damaged system. To this aim, it is useful to consider the following equilibrium equations:

$$\mathbf{K}_0\mathbf{s}_0 = \mathbf{f}_0 \qquad \mathbf{K}_1\mathbf{s}_1 = \mathbf{f}_1 \tag{4}$$

where the subscripts "0" and "1" refer to the intact state and the damaged state of the structure, respectively (Figure 1.a). Based on these equations, the displacement vector of the intact system $\mathbf{s}_0$ can be related to the displacement vector of the damaged system $\mathbf{s}_1$ as follows:

$$\mathbf{s}_0 = \mathbf{s}_1 + \mathbf{K}_1^{-1}\hat{\mathbf{f}}_1 = \mathbf{K}_1^{-1}(\mathbf{f}_1 + \hat{\mathbf{f}}_1) \tag{5}$$

$$\hat{\mathbf{f}}_1 = (\mathbf{K}_1 - \mathbf{K}_0)\mathbf{s}_0 - (\mathbf{f}_1 - \mathbf{f}_0) = \Delta\mathbf{K}\mathbf{s}_0 - \Delta\mathbf{f} \tag{6}$$

where $\hat{\mathbf{f}}_1$ is a vector of nodal forces equivalent to the effects of repair (Figure 1.b). This vector represents the additional nodal forces that must be applied to the damaged system to achieve the nodal displacements of the intact system, and it is called *backward pseudo-load vector*.

In a dual way, the displacement vector of the damaged system $\mathbf{s}_1$ can be related to the displacement vector of the intact system $\mathbf{s}_0$ as follows:

$$\mathbf{s}_1 = \mathbf{s}_0 + \mathbf{K}_0^{-1}\hat{\mathbf{f}}_0 = \mathbf{K}_0^{-1}(\mathbf{f}_0 + \hat{\mathbf{f}}_0) \tag{7}$$

$$\hat{\mathbf{f}}_0 = -(\mathbf{K}_1 - \mathbf{K}_0)\mathbf{s}_1 + (\mathbf{f}_1 - \mathbf{f}_0) = -\Delta\mathbf{K}\mathbf{s}_1 + \Delta\mathbf{f} \tag{8}$$

where $\hat{\mathbf{f}}_0$ is a vector of nodal forces equivalent to the effects of damage (Figure 1.c). This vector represents the additional nodal forces that must be applied to the intact system to achieve the nodal displacements of the damaged system, and it is called *forward pseudo-load vector*. Backward and forward pseudo-loads can be related as follows:

$$\mathbf{K}_0^{-1}\hat{\mathbf{f}}_0 + \mathbf{K}_1^{-1}\hat{\mathbf{f}}_1 = 0 \tag{9}$$

The concept of pseudo-loads can be usefully exploited to define, in a dual way, two energy-based indicators related to the structural performance of both the intact system and the damaged system. The first one of these indicators is the difference of stored energy Φ between the intact system (Φ_0) and the damaged system after the application of the backward pseudo-loads ($\hat{\Phi}_1$):

$$\begin{aligned}\Delta\Phi_0 = \Phi_0 - \hat{\Phi}_1 &= \frac{1}{2}\mathbf{s}_0^T\mathbf{f}_0 - \frac{1}{2}\mathbf{s}_1^T(\mathbf{f}_1 + \hat{\mathbf{f}}_1)\\ &= -\frac{1}{2}\mathbf{s}_0^T(\hat{\mathbf{f}}_1 + \Delta\mathbf{f})\end{aligned} \tag{10}$$

The area $\mathrm{OP_0\hat{P}_1}$ in Figure 1a represents the energy $\Delta\Phi_0$ for the case $\Delta\mathbf{f} = 0$. The second indicator is the difference of stored energy Φ between the intact system after the application of the forward pseudo-loads ($\hat{\Phi}_0$) and the damaged system (Φ_1):

$$\begin{aligned}\Delta\Phi_1 = \hat{\Phi}_0 - \Phi_1 &= \frac{1}{2}\mathbf{s}_0^T(\mathbf{f}_0 + \hat{\mathbf{f}}_0) - \frac{1}{2}\mathbf{s}_1^T\mathbf{f}_1\\ &= \frac{1}{2}\mathbf{s}_1^T(\hat{\mathbf{f}}_0 - \Delta\mathbf{f})\end{aligned} \tag{11}$$

The area $\mathrm{O\hat{P}_0P_1}$ in Figure 1a represents the energy $\Delta\Phi_1$ for the case $\Delta\mathbf{f} = 0$.

3 MEASURE OF STRUCTURAL ROBUSTNESS

3.1 *Robustness indices*

Structural robustness can be viewed as the ability of a system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself. According to this general definition, a measure of structural robustness should arise by comparing the structural performance of the system in the original state, in which the structure is fully intact, and in a perturbed state, in which a prescribed damage scenario is applied (Frangopol and Curley 1987, Biondini *et al.* 2008). Based on this approach, the performance indicators are used as state variables, and a direct measure of structural robustness is obtained through robustness

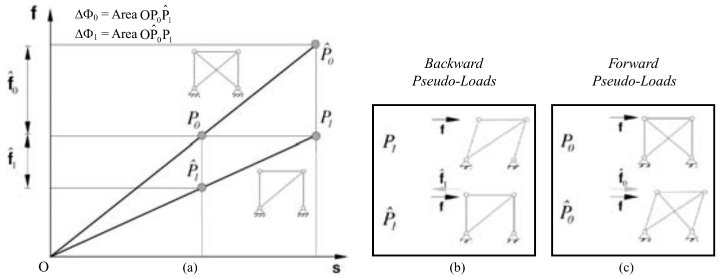

Figure 1. (a) Structural behavior of a truss system in the intact state and after elimination of one member. (b) Backward pseudo-loads (effects of repair). (c) Forward pseudo-loads (effects of damage).

indices, that are dimensionless functions of these variables varying in the range [0, 1]. In this study, the following robustness indices are considered:

$$\rho_d = \frac{d_1}{d_0} \qquad \rho_t = \frac{t_1}{t_0} \tag{12}$$

$$\rho_c = \frac{c_0}{c_1} \qquad \rho_T = \frac{T_0}{T_1} \tag{13}$$

$$\rho_s = \frac{\mathbf{s}_0}{\mathbf{s}_1} \qquad \rho_\Phi = \frac{\Phi_0}{\Phi_1} \tag{14}$$

$$\rho_0 = 1 - \frac{\Delta\Phi_0}{\Phi_0} \qquad \rho_1 = 1 - \frac{\Delta\Phi_1}{\hat{\Phi}_0} \tag{15}$$

The indices ρ_d, ρ_t, ρ_c, and ρ_T, are related to the properties of the structural system only, while the indices ρ_s, ρ_Φ, ρ_0, and ρ_1, take also the loading condition into account.

To discuss the effectiveness of these indices, the structural robustness of the truss system shown in Figure 2 is evaluated under the progressive damage of each one of its members. The cross-section of all members is circular with radius r. For the damaged member an external layer of uniform thickness t is removed. Therefore, the amount of damage can be specified by means of the damage index $\delta = t/r \in [0; 1]$. The results are shown in Figure 3. It can be noted that, globally, the robustness indices allow to evaluate the role played by each member on the overall performance of the damaged system. However, the following critical aspects can also be outlined:

– The index ρ_c may show an increase of robustness under damage evolution ($\rho_c > 1$).
– The index ρ_d is not able to catch the different role played by each element.
– The index ρ_t shows a very little sensitivity to damage.
– The indices ρ_t and ρ_0 are not able to identify the failure condition $\rho = 0$ for $\delta = 1$.

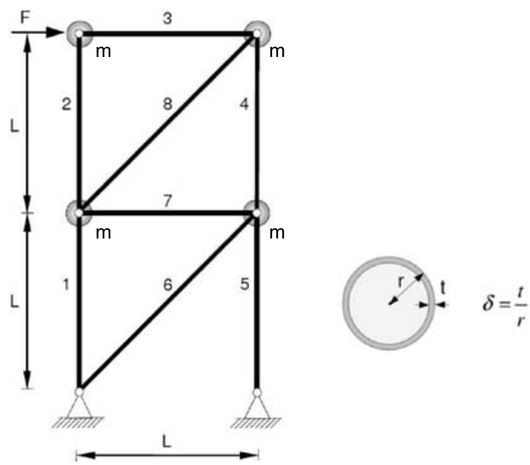

Figure 2. Truss system undergoing damage of one member.

Therefore, the indices ρ_c, ρ_d, ρ_t, and ρ_0, seem to be not suitable to fully describe the effects of damage on the structural performance. On the contrary, the indices ρ_t, ρ_s, ρ_Φ, and ρ_1, can provide a very effective measure of structural robustness.

3.2 *Structural integrity index*

A robustness index should be able to identify the structural collapse by assuming at failure the value $\rho = 0$. However, in robustness evaluations may also be crucial to quantify the severity of a structural failure with regards to its consequences. For example, the collapse of the whole structural system should be considered much more important than the collapse of a single member. For this reason, a possible importance measure of a structural failure could be provided by the following structural integrity index:

$$\rho_V = \frac{V_1}{V_0} \tag{16}$$

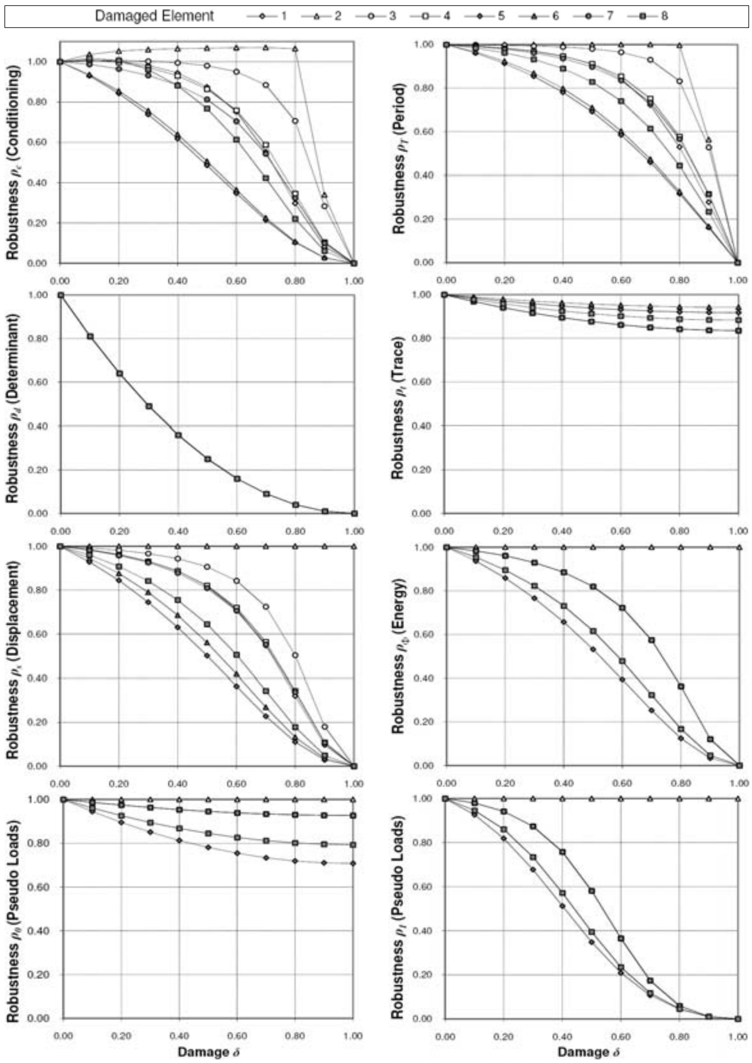

Figure 3. Truss system undergoing damage of one member. Robustness indices ρ versus damage δ.

Table 1. Structural integrity index for the truss in Figure 2.

Damaged element	Structural integrity index ρ_V
1	0.25
2	0.75
3	0.75
4	0.50
5	0.00
6	0.00
7	0.25
8	0.50

where V_1 is the portion of structural volume V_0 which remains intact after damage. It is worth noting that also members not directly exposed to damage may fail. For example, a complete damage of member 8 of the truss shown in Figure 2 causes also the failure of members 2, 3, and 4 ($\rho_V = 0.50$). Table 1 provides the structural integrity index associated to the failure of each single member. Failed members involved in a collapse mechanism can be identified based on the eigenvectors $\mathbf{s}_i$ of the stiffness matrix $\mathbf{K}$ associated with the eigenvalues $\lambda_i(\mathbf{K}) = 0$.

4 DAMAGE PROPAGATION

4.1 *Propagation mechanisms*

For redundant structures, local damage or failure of a member usually does not involve the collapse of the whole system. As a consequence, after failure of one member other members may fail, and the sequence of local failures propagates throughout the overall system until its collapse is reached. The mechanism of damage propagation is usually related to the causes of the damage itself. In Restelli (2007) two alternative propagation mechanisms, defined as directionality-based and adjacency-based, have been investigated.

In the directionality-based mechanism, damage propagates along the direction normal to the axis of the first failed member. For example, with reference to the frame system shown in Figure 4.a, the damage of member 1 is followed in sequence by the damage of elements 2, 3, and 4. The directionality-based mechanism is typical of damage induced by severe loadings, like explosions or impacts, which generally tends to propagate along the direction of loading.

In the adjacency-based mechanism, damage propagates towards the members directly connected with other members already damaged. For example, with reference to the frame system shown in Figure 4.b, the damage of element 1 can be followed by the damage of the elements 5 and 13. The adjacency-based mechanism is typical of damage induced by aggressive agents, which generally tends to propagate through the structure based on diffusion processes.

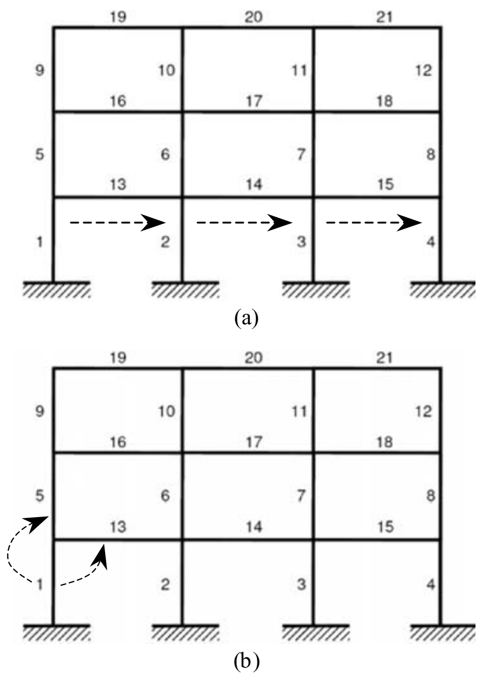

Figure 4. Mechanisms of damage propagation. (a) Directionality-based mechanism. (b) Adjacency-based mechanism.

4.2 *Fault-tree analysis*

Starting from the local definition of damage, and based on a prescribed propagation mechanism, a damage scenario at the system level can be developed by using a damage-sensitive fault-tree analysis. In such a way, all the feasible damage paths associated with the propagation mechanism and the actual topology of the system can be described by branched networks where the level of activation of each nodal connection is properly tuned to account for the prescribed amount of local structural damage (see Restelli 2007).

To describe the main features of this approach, the structural robustness of the frame system shown in Figure 5.a is evaluated under a distribution of lateral loads. The cross-sections of beams and columns, as well as the assumed cross-sectional damage patterns, are shown in Figures 5.b and 5.c. The fault-tree analysis is carried out by assuming an adjacency-based propagation mechanism and a damage level $\delta = 1$ for each element. The results are represented in Figure 6 in terms of countoured branched network for the displacement-based robustness index ρ_s.

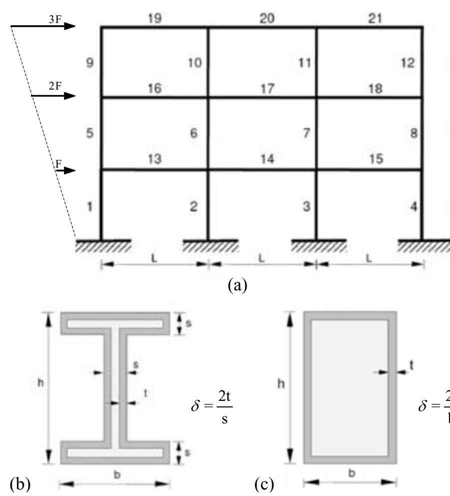

Figure 5. Frame system under damage. (a) Geometry ($L/H = 2$), structural scheme, and loading. Cross-sections of (b) beams and (c) columns ($h/b = 1.5$; $h/s = 15$).

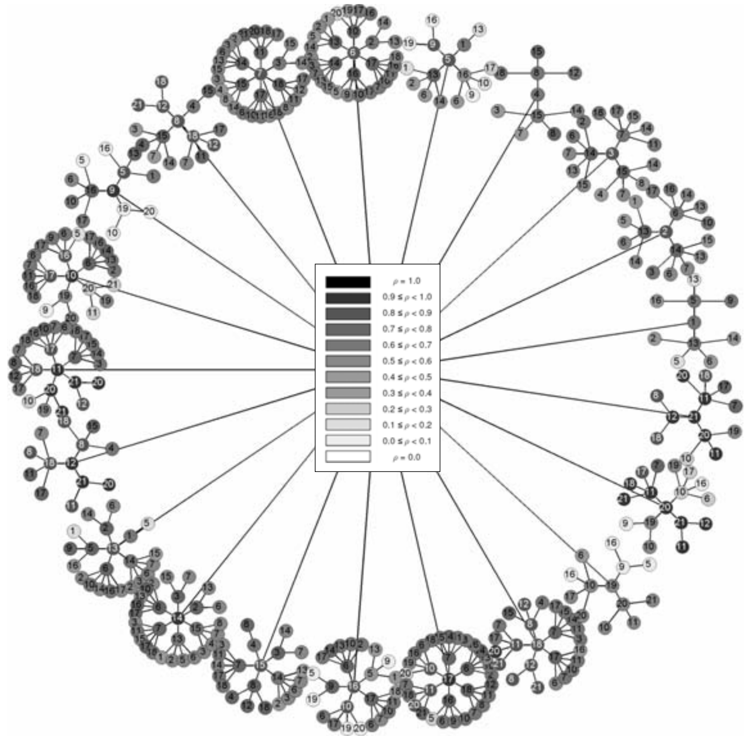

Figure 6. Fault-tree analysis of a frame system undergoing damage with adjacency-based propagation mechanism. Contoured branched network of structural robustness (displacement-based robustness index ρ_s) for a damage level $\delta = 1$.

This mapping provides a comprehensive vision and a quantitative measure of the structural resources of the system with respect to all the considered damage propagation paths.

5 CONCLUSIONS

A general approach aimed to define a measure of structural robustness of deteriorating structural systems by means of a set of dimensionless indices has been presented. An index of structural integrity aimed to quantify the severity of a structural failure with regards to its consequences, has been also proposed. Moreover, a model of damage propagation at the system level has been developed by considering two alternative propagation mechanisms, defined as directionality-based mechanism and adjacency-based mechanism. In this way, all the feasible damage paths associated with the actual topology of the structural system have been identified by a damage-sensitive fault-tree analysis and described by branched networks. The presented applications demonstrated the effectiveness of the proposed measures of structural robustness, as well as of the adopted damage propagation criteria, and highlighted the potentialities of the proposed approach in the context of a robust structural design.

ACKNOWLEDGEMENT

This study is supported by research funds PRIN2005 (prot. 2005082490) from the Italian Ministry of University and Research—Department of Structural Engineering, Politecnico di Milano.

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