On Structural Robustness, Redundancy and Static Indeterminacy

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ABSTRACT

The effects of prescribed damage scenario on robustness of structural systems with different degrees of static indeterminacy are investigated. Damage is viewed as a progressive deterioration of the material properties and its amount is specified at the member level by means of a damage index associated with prescribed patterns of cross-sectional deterioration. The variation of meaningful parameters of the structural response with respect to the values associated with the undamaged system are used to formulate dimensionless measures of structural robustness. The differences among robustness, redundancy and static indeterminacy are pointed out. Truss systems are then analyzed to investigate their robustness.

INTRODUCTION

During the last few decades, progressively increasing attention has been focused on the concepts of structural robustness and structural redundancy. The first developments in this field followed the partial collapse in 1968 of the Ronan Point high rise building in London after a relatively small gas explosion. More recently, other building collapse events (including the attacks on the Alfred P. Murrah building in 1995 and on the twin towers of the World Trade Center in 2001) emphasize the need for additional research towards the development of new concepts and methods in this field. As a consequence of these and other recent dramatic structural failures, the importance of reliable design procedures leading to conceive redundant and robust structures is nowadays widely recognized.

The terms robustness and redundancy and static indeterminacy are often used as synonymous. However, they denote different properties of the structural system. In fact, structural robustness can be viewed as the ability of the system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself. Structural redundancy can instead be defined as the ability of the system to redistribute among its members the load which can no longer be sustained by some other damaged members Redundancy is usually associated with the degree of static indeterminacy. However, it has been demonstrated that the degree of static indeterminacy is not a consistent measure for structural redundancy (Frangopol and Curley 1987). In fact, structures with lower degrees of static indeterminacy can have a greater redundancy than structures with higher degrees of static indeterminacy. It has been shown, that structural redundancy depends on many factors, such as structural topology, member sizes, material properties, applied loads and load sequence, among others (Frangopol and Curley, 1987).

In this paper, the effects of prescribed damage scenario on the robustness of systems with various degrees of indeterminacy are investigated. The attention is focused on truss structures and damage is viewed as a progressive deterioration of the material properties and its amount is specified at the member level by means of a damage index associated with prescribed patterns of cross-sectional deterioration. After a damage scenario is defined, the deterioration effects on the system performance are evaluated with reference to suitable performance indicators identified with meaningful parameters of the structural response. The variation of these indicators with respect to the values associated with the performance of the undamaged system are used to formulate dimensionless measures of structural robustness (Restelli 2007). Truss systems are then analyzed under prescribed damage to investigate their robustness.

MEASURE OF STRUCTURAL ROBUSTNESS

The concept of robust structures is still an issue of controversy since there are no well established and generally accepted criteria for a consistent definition and a quantitative measure of structural robustness. As previously pointed out, structural robustness can be viewed as the ability of the system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself. According to this definition, a measure of structural robustness should arise by comparing the structural performance of the system in the original state, in which the structure is fully intact, and in a perturbed state, in which a prescribed damage scenario is applied.

This approach is used in Frangopol and Curley (1987) to evaluate the effects of damage on the overall collapse load of truss structures formed by brittle or ductile members. Strength and ductility, as well as other performance indicators associated with ultimate conditions, are of great importance in robustness evaluations associated with damage suddenly provoked by accidental actions, like explosion or impacts. However, damage could also arise slowly in time from aging of structures, as induced for example by environmental aggressive agents. In this context, performance indicators associated with serviceability conditions, like stiffness and first yielding, may become of major importance in life-cycle robustness evaluations.

The effectiveness of several performance indicators in evaluating structural robustness has been investigated in Restelli (2007). They include indicators associated with the properties of the structural system only, like eigenvalues and conditioning number of the overall stiffness matrix, and indicators also depending on the loading scenario, like stored energy, displacements, and vectors of nodal forces equivalent to the effects of damage (backward or forward pseudo-loads). All these indicators could be adopted as state variables affecting the robustness of a structural system. A direct measure of structural robustness within the range [0, 1] is then obtained through functions of such variables, that are robustness indices.

A set of dimensionless robustness indices has been introduced in Restelli (2007). In the present study, the following index associated with the displacements of the system is considered:

$$\rho = \frac{\| \mathbf{s}_0 \|}{\| \mathbf{s}_d \|} \tag{1}$$

where $\mathbf{s}$ is the displacement vector, $\| \cdot \|$ denotes the euclidean scalar norm, and the subscripts "0" and "d" refer to the intact and damaged state of the structure, respectively.

DAMAGE MODELING

In view of a life-cycle robust design able to cover the large amount of uncertainty associated with aging processes, it is of great interest to develop suitable measures of structural robustness with respect to a progressive deterioration of the structural performance. Therefore, structural damage is here viewed as a progressive deterioration of the material properties and its amount is specified at the member level by means of a damage index $0 \leq \delta \leq 1$ associated with prescribed patterns of cross-sectional deterioration. Focusing the attention on truss structures, damage must be related to a progressive deterioration of the cross-sectional area, or $A_d = \alpha A_0$, with $0 \leq \alpha \leq 1$. Figure 1 shows the relationship $\alpha = \alpha(\delta)$ for circular cross-sections, ① solid and ② hollowed, undergoing uniform damage along the external boundary.

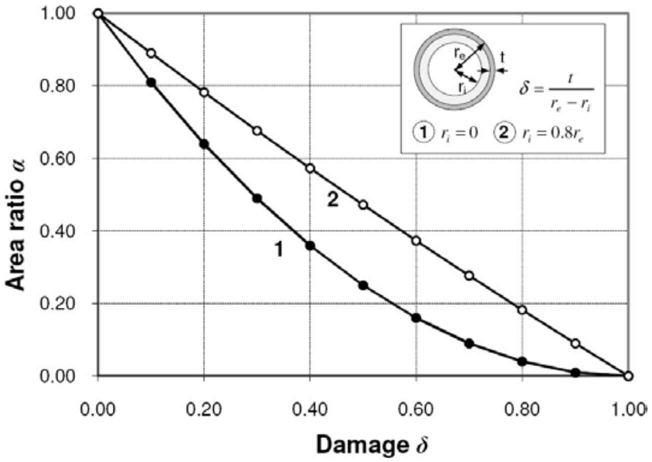


FIGURE 1 – AREA RATIO $\alpha = A_d/A_0$ VERSUS DAMAGE INDEX δ FOR CIRCULAR CROSS-SECTIONS, ① SOLID AND ② HOLLOWED, UNDERGOING UNIFORM DAMAGE ON EXTERNAL BOUNDARY.

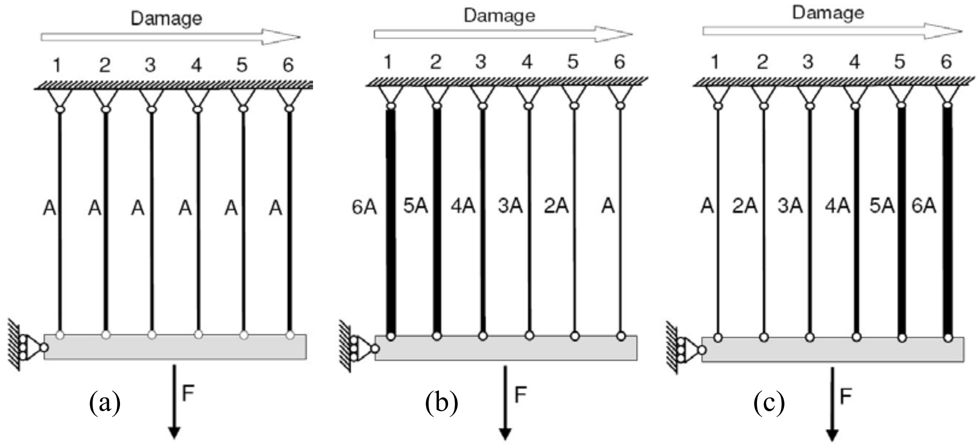
STRUCTURAL ROBUSTNESS AND PROGRESSIVE COLLAPSE

Local damage or failure of a member usually involves a redistribution of the internal forces among the other members of the structural system. As a consequence, if the amount of redistributed forces is large enough, other members may fail and the sequence of local failures may propagate throughout the overall system until its collapse is reached. A possible way to avoid this type of progressive collapse is to design robust structures for which alternate load paths are possible and the most critical members are properly protected from accidental or environmental damage.

To highlight the role of robustness on progressive collapse, a preliminary investigation is developed with reference to the simple parallel systems composed of $n = 6$ truss members shown in Figure 2 (Restelli 2007). The force F_k carried by each member $k = 1,2,\ldots,6$, is a portion ν_k of the total applied load F in such a way that the equilibrium is satisfied:

$$\nu_k = \frac{F_k}{F} \;, \qquad \sum_1^n \nu_k = 1 \tag{2}$$

Due to the static indeterminacy of the problem, the coefficients ν_k depend on the geometrical and mechanical properties of the members, as well as on the damage state of the system.



(a) (b) (c)

FIGURE 2 – PARALLEL SYSTEMS UNDERGOING DAMAGE OF ALL MEMBERS.

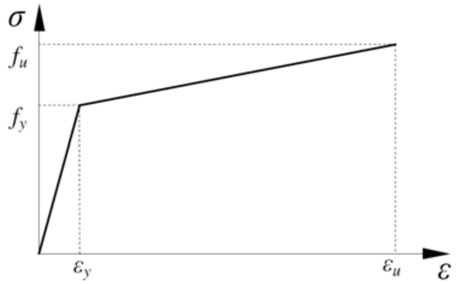


FIGURE 3 – CONSTITUTIVE LAW OF THE MATERIAL.

All members are assumed to have circular cross-section with uniform damage along the external boundary, as shown in Figure 1 for cross-section 1. For each member k the deterioration of the cross-sectional area is described by the corresponding damage index $0 \le \delta_k \le 1$. Damage is assumed to develop in each member and proceed from a member k to the adjacent one (k+1) in a progressive and continuous way. Based on this assumption, the damaged state of the system can be described by a total cumulative damage function $0 \le \Delta_k \le n$ defined as follows:

$$\Delta_k = \sum_{i=1}^{k} \delta_i = (k-1) + \delta_k \tag{3}$$

Three cases are studied: (a) all bars k have the same initial area A_k=A (Figure 2.a); (b) each bar k has initial area A_k=(n–k+1)A, in such a way that damage proceeds from the strongest member to the weakest one (Figure 2.b); (c) each bar k has initial area A_k=kA, in such a way that damage proceeds from the weakest member to the strongest one (Figure 2.c). For all cases the material behavior is described by a bilinear constitutive law with hardening, as shown in Figure 3, with overstrength ratio $f_u/f_y \cong 1.5$ and ductility $\varepsilon_u/\varepsilon_y \cong 10$.

As damage increases, the robustness ρ of the system changes and a redistribution of the internal forces ν_k occurs. The evolution of this process depends on the ratio η between the applied load F and the load F_y associated with the first yielding of the system:

$$\eta = \frac{F}{F_y} \tag{4}$$

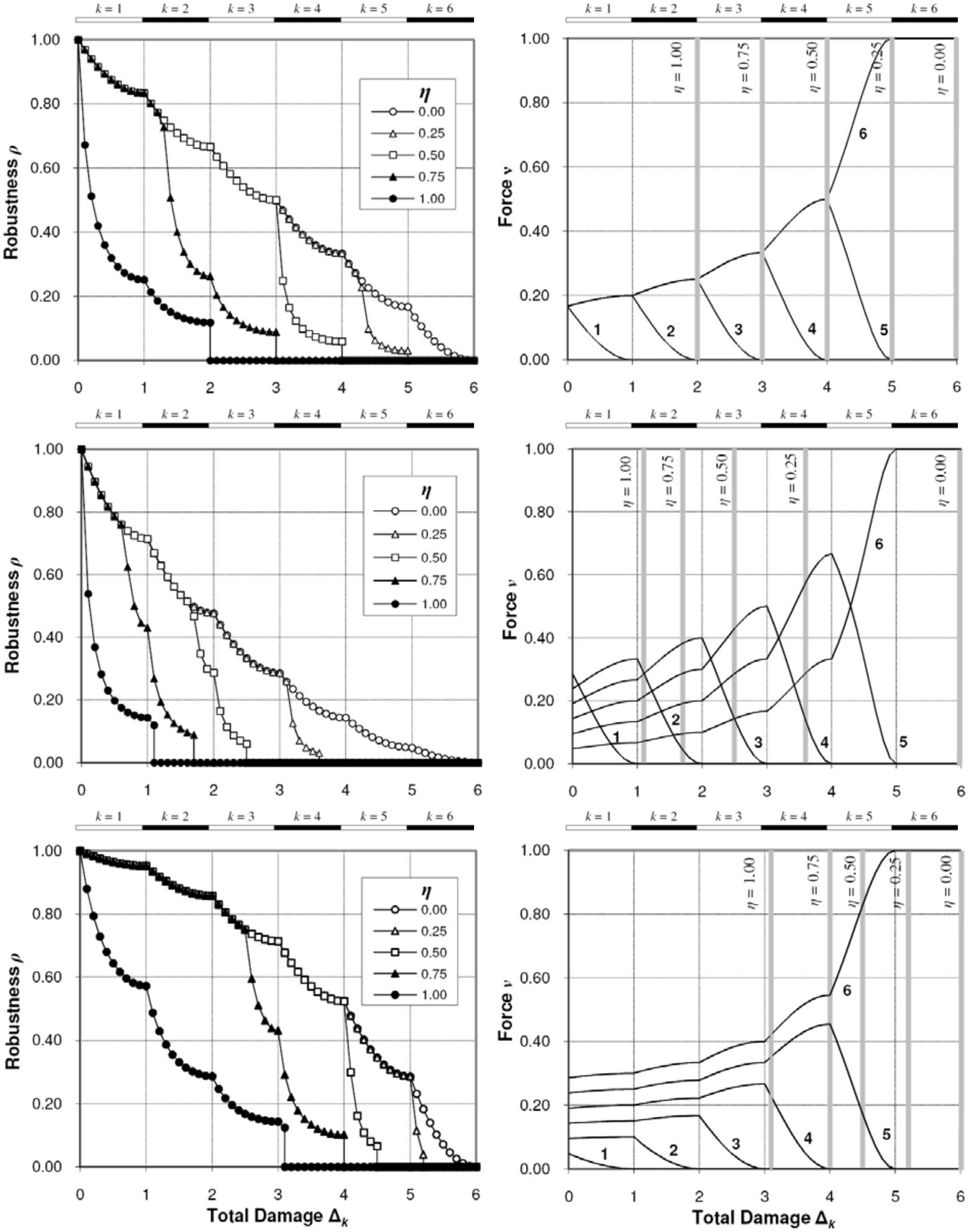


FIGURE 4 – PARALLEL SYSTEMS UNDERGOING DAMAGE OF THEIR MEMBERS (DAMAGE PROCEEDS FROM MEMBER *k*=1 TO MEMBER *k*=6): ROBUSTNESS INDEX ρ AND INTERNAL FORCES ν_K VERSUS THE CUMULATIVE DAMAGE Δ_K FOR DIFFERENT LEVELS OF THE LOAD RATIO η.

Structural collapse is reached when the propagation of damage leads to failure of all members. In this limit condition robustness vanishes and total damage is identified by the following threshold:

$$\Delta_{k,c} = \min\left\{ \Delta_k \;\middle|\; \rho(\Delta_k) = 0 \right\} \tag{5}$$

Therefore, the functions $\rho=\rho(\Delta_k)$ and $\nu=\nu(\Delta_k)$ with $\Delta_k \leq \Delta_{k,c}$ define the paths followed by the system towards its progressive collapse. Figure 4 shows the evolution of the robustness index ρ and of the internal forces ν_k as a function of the cumulative damage Δ_k for different levels of the load ratio η and for each one of the three case studied. These results can be used to check if a progressive collapse occurs under prescribed loading and damaging scenarios or, conversely, to evaluate the limit load and/or the damage threshold associated with the occurrence of progressive collapse.

It should be noted that the results of case (a) are intermediate with respect to the results of case (b) and case (c). For case (b) damage starts in the strongest members, which progressively exchange their leading role with the weakest members. Consequently, this case is characterized by the lower robustness and it is more prone to reach a progressive collapse. On the contrary, for case (c) damage starts in the weakest members and the leading role of the strongest members can be fully exploited until collapse. Therefore, the configuration in case (a) should be considered as the best one for a robust design, unless there are reasons for considering one direction of damage propagation more probable than others. More generally, it can be concluded that very strong members playing a disproportionate role in the structural system should be avoided in design of robust structures. And when this is not possible, adequate remedy should be adopted to properly protect the most important members against any occurrence of damage.

STRUCTURAL ROBUSTNESS AND STATIC INDETERMINACY

Structural redundancy is the ability of a system to redistribute among its members the load which can no longer be sustained by some other members due to their damage. Therefore, redundancy represents a key factor for structural robustness and progressive collapse. Redundancy is usually associated with the degree of static indeterminacy. However, it has been demonstrated that the degree of static indeterminacy is not a consistent measure for structural redundancy (Frangopol and Curley 1987). In fact, structures with lower degrees of static indeterminacy can have a greater redundancy than structures with higher degrees of static indeterminacy. It has been shown, that structural redundancy depends on many factors, such as structural topology, member sizes, material properties, applied loads and load sequence, among others (Frangopol and Curley, 1987). As an example, consider the simple n=6 parallel systems shown in Figure 5 (Restelli 2007).

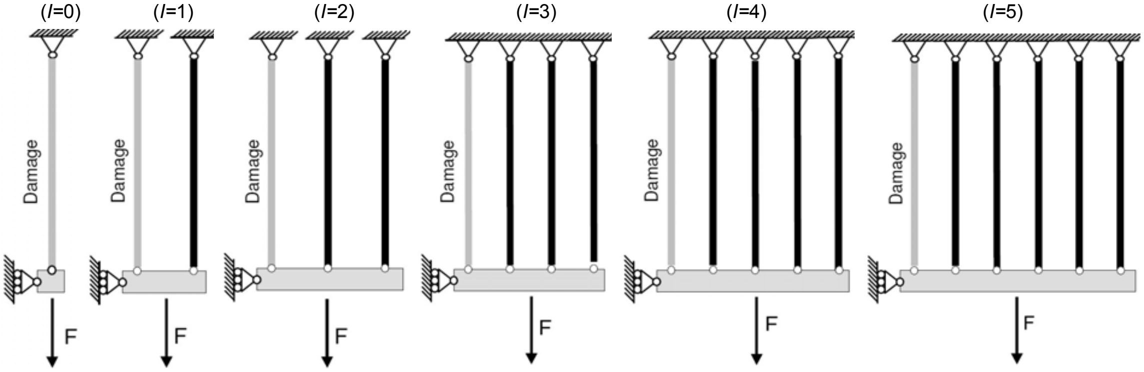


FIGURE 5 – PARALLEL SYSTEMS UNDERGOING DAMAGE OF ONE MEMBER.

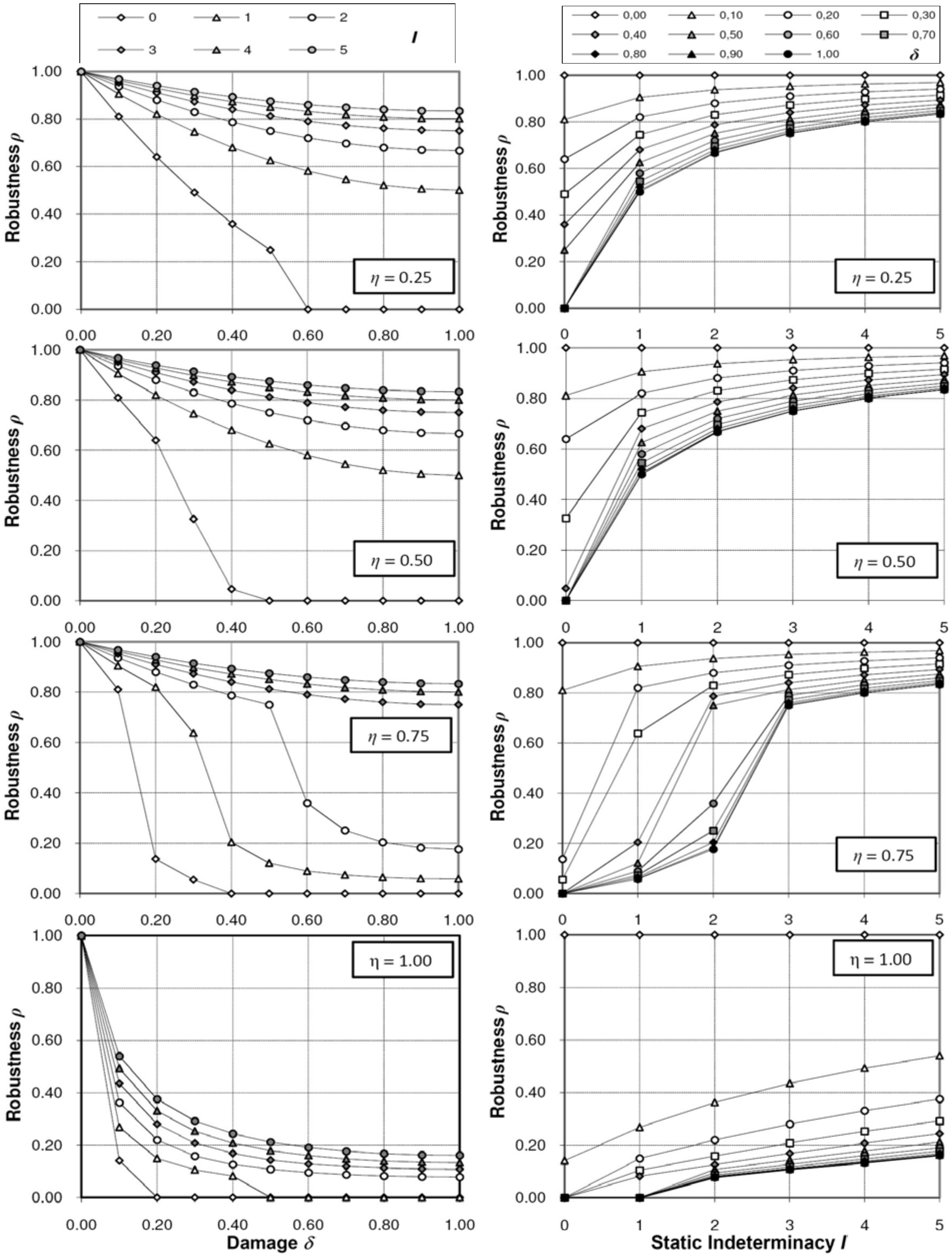


FIGURE 6 – PARALLEL SYSTEMS UNDERGOING DAMAGE OF ONE MEMBER: ROBUSTNESS INDEX ρ VERSUS DAMAGE δ AND INDETERMINACY I FOR DIFFERENT LEVELS OF THE LOAD RATIO η.

For each k-bar system, with $k=1,2,\ldots,6$, the degree of static indeterminacy is $I=(k-1)$. All members are identical and their cross-sectional shape and material behavior are the same as in the previous example. Damage is assumed to develop in one member only and its evolution is described by the corresponding damage index $0\leq\delta\leq 1$.

Figure 6 shows the evolution of the robustness index ρ as a function of the damage index δ (Figure 6.a) and of the degree of static indeterminacy I (Figure 6.b), for different values of the load ratio η. These results show that robustness increases as static indeterminacy increases. However, it is worth noting that only a certain degree of static indeterminacy (i.e. $I\leq 2$) provides a significant contribution to structural robustness for all load ratio, and this contribution becomes more important as much as damage increases. For the higher degree of static indeterminacy (i.e. $I\geq 3$) its positive role is in general less important, and the contribution to structural robustness tends to be significant only for severe damage and very high values of the load ratio. Clearly, higher levels of static indeterminacy would be required when more than one member is affected by damage. In general, it can be concluded that in design of robust structures the degree of static indeterminacy should be adequately allocated as a function of the expected amount of damage.

In the previous example robustness increases as static indeterminacy increases. However, this result cannot be generalized, since an increase in the degree of static indeterminacy does not necessarily lead to an increase of robustness. Consider for example the $n=4$ mixed series-parallel truss systems shown in Figure 7 (Restelli 2007). The degree of static indeterminacy of each system $k=1,2,\ldots,4$, is $I=2\times(k-1)$. All members are identical with hollowed circular cross-section and uniform damage along the external boundary, as shown in Figure 1 for cross-section 2. The material behavior is the same as in the previous examples. Damage is assumed to simultaneously develop in the two adjacent members located at the bottom of the truss beam at the middle span, and its evolution is described by the corresponding damage index $0\leq\delta\leq 1$.

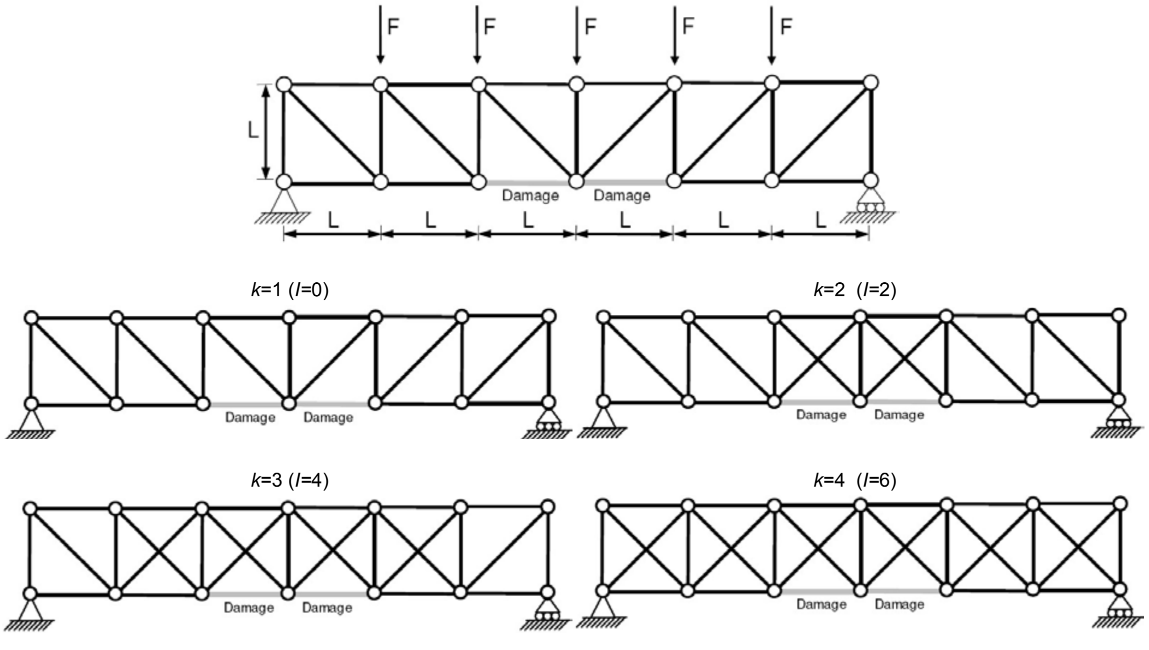


FIGURE 7 – MIXED SERIES-PARALLEL TRUSS SYSTEMS UDERGOING DAMAGE OF TWO MEMBERS.

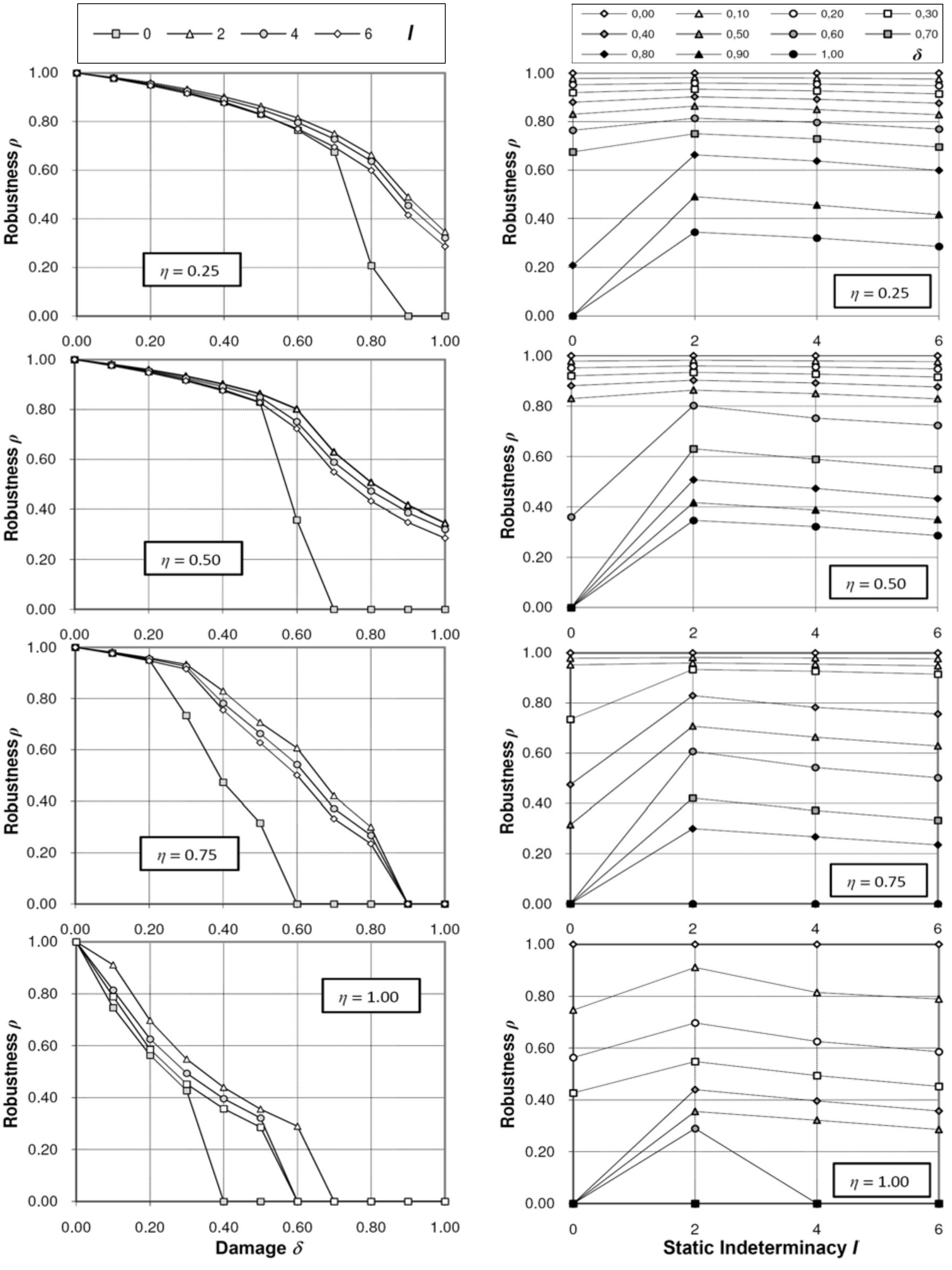


FIGURE 8 – TRUSS SYSTEMS UNDERGOING DAMAGE OF TWO MEMBERS: ROBUSTNESS INDEX ρ VERSUS DAMAGE δ AND STATIC INDETERMINACY I FOR DIFFERENT LEVELS OF THE LOAD RATIO η.