

Lifetime Optimization of Deteriorating Structures

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ABSTRACT: A consistent approach to the optimal design of durable structures must consider the time evolution of the structural performance induced by the progressive deterioration of the system properties. To comply with this requirement, a new conceptual approach to the minimum lifetime cost design of deteriorating structures under multiple loading conditions is presented. The proposed procedure is applied to the lifetime structural optimization of a steel truss and of a reinforced concrete frame under different damage and maintenance scenarios. The obtained results highlight the fundamental role played by both the time-variant performance and the maintenance planning in the selection of the optimum structural design.

1 INTRODUCTION

For structures exposed to damaging environments the structural performance must be considered as time-dependent, mainly because of the progressive deterioration of the mechanical properties of materials which makes the structural system less able to withstand the applied actions. Therefore, a consistent approach to optimum design of durable structures should lead to find design solutions able to comply with the desired performance not only at the initial time of construction, but also during the whole expected service lifetime by taking into account the effects induced by unavoidable sources of mechanical damage and by eventual maintenance interventions.

This paper presents a new conceptual approach to the lifetime optimization of deteriorating structures under multiple loading conditions (Azzarello et al. 2006, Biondini and Marchiondelli 2006a). The structural damage is modeled by introducing a proper material degradation law and the structural analysis is carried out at different time instants in order to assess the time evolution of the system performance. The design constraints are related to both the time-variant stress and displacement state, as well as to the amount of structural damage. The objective function is formulated by accounting for both the initial cost of the structure and the costs of possible maintenance interventions, that are properly discounted over time and assumed to be proportional to the actual level of structural damage. The proposed procedure is initially presented for truss and framed structures made by homogeneous members. Finally, it is extended to the case of reinforced concrete structures by adopting different damage rates and material costs for both concrete and steel.

The lifetime structural optimization of a steel truss and of a reinforced concrete frame under different damage and maintenance scenarios proves the effectiveness of the proposed formulation. In particular, the obtained results highlight the fundamental role played by both the time-variant performance and the maintenance planning in the selection of the optimum structural design.

2 MODELING OF STRUCTURAL DAMAGE

With reference to truss and framed structures, damage is considered to affect the cross-sectional area $A=A(t)$, the elastic modulus $E=E(t)$, and the material strength $\bar{\sigma} = \bar{\sigma}(t)$ of each member:

$$A(t) = [1 - \delta_A(t)]A_0 \quad E(t) = [1 - \delta_E(t)]E_0 \quad \bar{\sigma}(t) = [1 - \delta_{\bar{\sigma}}(t)]\bar{\sigma}_0 \quad (1)$$

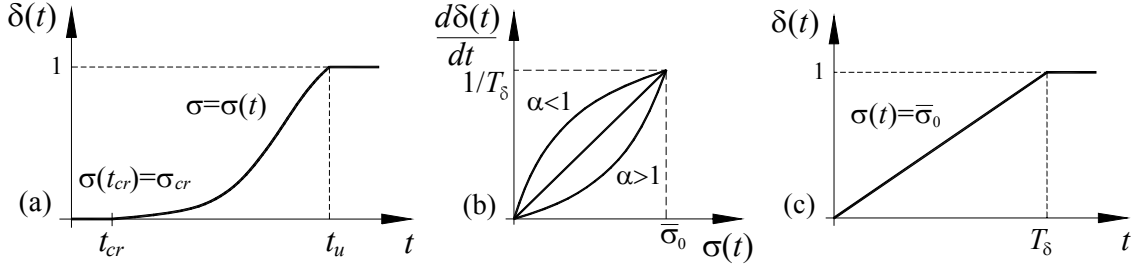


Figure 1. Modeling of structural damage. (a) Time evolution of the damage index $\delta=\delta(t)$. (b) Linear relationship between the rate of damage and the stress level $\sigma=\sigma(t)$. (c) Meaning of the damage parameter T_δ .

where $\delta_A, \delta_E, \delta_{\bar{\sigma}}$, are dimensionless damage indices which provide a direct measure of the damage level within the range [0; 1]. Proper correlation laws may be introduced to define the corresponding variation of other geometrical properties of the cross-section, like the inertia moment, etc.

The time evolution of the damage indices $\delta_A, \delta_E, \delta_{\bar{\sigma}}$, clearly depends on the physics of the deterioration process, usually related also to the stress state $\sigma=\sigma(t)$ (Figure 1.a). Therefore, a reliable assessment of the decreasing structural performance during time requires the formulation of deterioration models suitable to describe the actual damage evolution and its interaction with the structural behavior (Biondini *et al.* 2004). However, despite the inherent complexity of damage laws, very simple degradation models could be successfully adopted in order to define an effective hierarchical classification of the design alternatives (Biondini and Marchiondelli 2006b).

Without any loss of generality, in this study it is assumed that all material properties undergo the same damage process, or $\delta_A = \delta_E = \delta_{\bar{\sigma}} \equiv \delta$. In addition, the damage index $\delta=\delta(t)$ is correlated to the time-variant structural behavior by assuming the following relationship between the rate of damage and the acting stress $\sigma=\sigma(t)$ (Figure 1.b):

$$\frac{d\delta(t)}{dt} = \frac{1}{T_\delta} \left[\frac{\sigma(t)}{\bar{\sigma}_0} \right]^\alpha \quad \bar{\sigma}_0 = \begin{cases} \bar{\sigma}_0^+ & \text{if } \sigma \geq 0 \\ \bar{\sigma}_0^- & \text{if } \sigma < 0 \end{cases} \quad (2)$$

where $\alpha \geq 0$ is a suitable constant, $\bar{\sigma}_0^-$ and $\bar{\sigma}_0^+$ are the minimum and maximum allowable stress at the initial time $t = t_0$, respectively, and T_δ represents the time period required for a complete damage under a constant stress level $\sigma(t) = \bar{\sigma}_0$ (Figure 1.c). In addition, the initial condition $\delta(t_{cr})=0$ with $t_{cr} = \max\{t \mid \sigma(t) \leq \sigma_{cr}\}$ is assumed, where $\sigma_{cr} \leq \bar{\sigma}_0$ is a critical stress threshold.

The index $\delta=\delta(t)$ fully describes the damage evolution in each point of the structure. However, due to its *local* nature, it does not seem handy for design purposes. A more synthetic *global* measure of damage $\bar{\delta} = \bar{\delta}(t)$ may be easily computed as the average of δ over the volume of the whole structural system (Biondini 2004, Azzarello *et al.* 2006).

3 LIFETIME OPTIMALITY CRITERIA

3.1 Structural Cost

Several quantities able to represent the structural performance may be chosen as targets for the optimal design. In the following, the adopted design target is the total cost C of the structure over its service lifetime, given by the sum of the initial cost C_0 and maintenance cost C_m :

$$C = C_0 + C_m \quad (3)$$

The initial cost C_0 is computed as follows:

$$C_0 = cV_0 \quad (4)$$

where V_0 is the total volume of material and c is the corresponding unit cost. The maintenance cost C_m can be evaluated by summing the costs of the individual interventions:

$$C_m = \sum_{k=1}^r \frac{C_m^k}{(1+\nu)^{(t_k-t_0)}} \quad (5)$$

where the cost C_m^k of each intervention $k=1, \dots, r$ has been referred to the initial time t_0 by taking a proper discount rate ν into account (Kong and Frangopol 2003).

3.2 Maintenance Scenario

The previous general formulation is now specialized to a prescribed maintenance scenario. In this scenario an essential maintenance aimed to totally restore the initial structural performance is performed after each design period T_D , or at each time instant $t_k = (t_0 + kT_D)$. In this way, all the interventions have the same cost $C_m^k = C_m^1$, and the total number of interventions applied during a prescribed service lifetime T_S is $r = \lceil (T_S/T_D) - 1 \rceil$. Therefore, the cost of maintenance is:

$$C_m = C_m^1 \sum_{k=1}^r \frac{1}{(1+v)^{kT_D}} = C_m^1 q \quad (6)$$

where the factor $q = q(T_S, T_D, v) \leq r$ depends on the prescribed parameters T_S , T_D , and v only.

The cost C_m^1 of the single intervention is related to the level of structural damage developed during each design period T_D . Since damage is not recovered during time, a measure of this damage is given by the global damage index $\tilde{\delta} = \delta(t)$ evaluated at the end of each design period T_D , or:

$$\tilde{\delta} = \hat{\delta}(t_k) = \hat{\delta}(t_{k-1} + T_D), \quad k=1, \dots, r \quad (7)$$

Based on the lifetime global damage index $\tilde{\delta}$, the following linear relationship is assumed:

$$C_m^1 = C_0 \tilde{\delta} \quad (8)$$

and the total lifetime structural cost C is finally formulated as follows:

$$C = C_0(1 + \tilde{\delta}q) \quad (9)$$

3.3 Role of Maintenance Cost

To highlight the actual role played by a prescribed maintenance program, the design of the tensioned bar shown at the top of Figure 2 is investigated. By denoting with d_0 the diameter of the undamaged cross-section, the total cost of the bar over the prescribed service lifetime T_S is:

$$C = C_0(1 + \tilde{\delta}q) = cA_0L(1 + \tilde{\delta}q) = c \frac{\pi d_0^2 L}{4} (1 + \tilde{\delta}q) \quad (10)$$

with $\tilde{\delta} = \tilde{\delta}(d_0)$. The diameter d_0 must be chosen in such a way that the acting stress $\sigma = \sigma(t)$ is no larger than the admissible stress $\bar{\sigma} = \bar{\sigma}(t)$ over the prescribed design period T_D :

$$\sigma(t) = \frac{F}{A(t)} = \frac{F}{A_0[1 - \delta(t)]} = \frac{4F}{\pi d_0^2[1 - \delta(t)]} \leq \bar{\sigma}(t) = \bar{\sigma}_0[1 - \delta(t)] \quad (11)$$

with $\delta(t) = \delta(d_0, t)$. In case damage is not considered ($\delta = \tilde{\delta} = 0$), the minimum cost solution d_0^* is simply given by the minimum diameter d_0 which satisfies the stress constraint:

$$d_0^* = d_{0,\min} = \sqrt{\frac{4F}{\pi \bar{\sigma}_0}} \quad C^* = C(d_0^*) = C_0(d_0^*) = c \frac{FL}{\bar{\sigma}_0} \quad (12)$$

On the contrary, when damage is properly included in the design problem, the minimum cost solution d_0^* is, in general, no longer associated with the diameter $d_{0,\min}$. In fact, higher d_0^* -values may be required to achieve a balance between the maintenance cost and the amount of damage.

These aspects are highlighted in Figure 2, where both the cost and structural efficiency of the bar versus its diameter d_0 are shown for different values of the damage rate $\theta = T_S/T_D$, with $\sigma_{cr}=0$, $\alpha=1$, and with reference to the following case study: $F=70$ kN, $\bar{\sigma}_0=100$ MPa, $T_S=100$ years, $T_D=10$ years, and $v=0$ ($q=r$). In particular, the diagrams shown in Figure 2 refer to the following quantities:

$$\chi_0^* = C_0/C^* \quad \chi_m^* = C_m/C^* \quad \chi^* = \chi_0^* + \chi_m^* \quad \rho = \sigma(T_D)/\bar{\sigma}(T_D) \quad (13)$$

where C^* denotes the optimal cost without damage ($\theta=0$). The following remarks can be made:

- The minimum feasible diameter without damage is $d_{0,\min}=29.9$ mm. Its value increases with θ .
- The initial cost χ_0^* increases and the maintenance cost χ_m^* decreases when d_0 increases. For a given value of d_0 , the maintenance cost χ_m^* increases with θ .
- The total cost χ^* has a minimum for $d_0^* \geq d_{0,\min}$, and the optimal diameter d_0^* increases with θ .
- The stress ratio ρ decreases with d_0 and increases with θ . The optimal solution d_0^* at the end of the design period T_D may be not fully stressed ($\rho \leq 1$).

Similar results are obtained by varying the parameter T_D (Azzarello 2005).

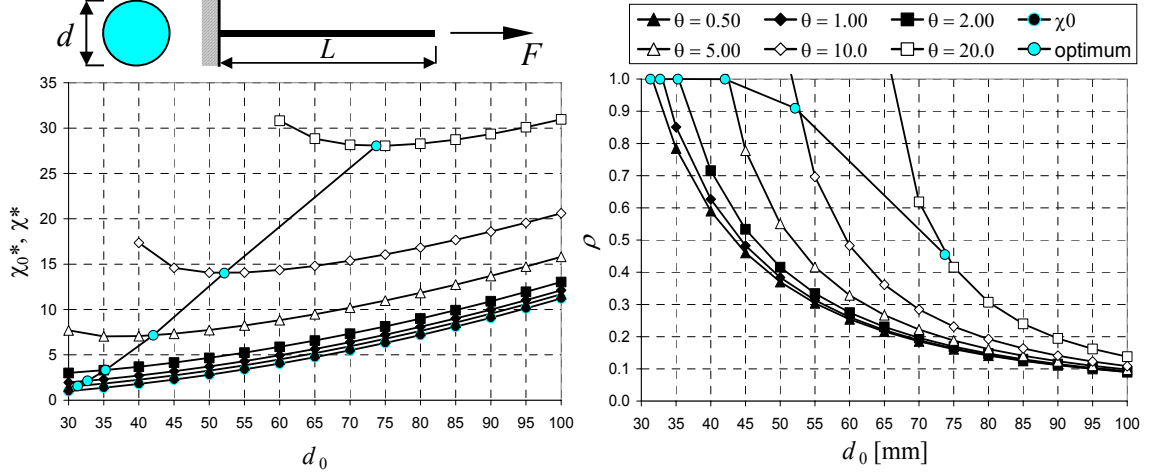


Figure 2. Lifetime cost and structural efficiency of a tensioned bar undergoing damage.

4 LIFETIME STRUCTURAL OPTIMIZATION

4.1 Formulation of the Optimization Problem

The purpose of a one-target lifetime design process is to find a vector of design variables $\mathbf{x} \in \mathcal{R}^n$ which optimizes the value of an objective function $f(\mathbf{x})$, according to both side constraints with bounds \mathbf{x}^- and \mathbf{x}^+ , and inequality time-variant behavioral constraints $\mathbf{g}(\mathbf{x}, t) \leq \mathbf{0}$:

$$\min_{\mathbf{x} \in D} f(\mathbf{x}) \quad D = \{ \mathbf{x} \mid \mathbf{x}^- \leq \mathbf{x} \leq \mathbf{x}^+, \mathbf{g}(\mathbf{x}, t) \leq \mathbf{0} \} \quad (14)$$

Based on the previously introduced cost concepts, the objective function $f(\mathbf{x})$ to be minimized is related to the total lifetime cost C of the structure as follows:

$$f(\mathbf{x}) \equiv C(\mathbf{x})/c = V_0(\mathbf{x})[1 + \tilde{\delta}(\mathbf{x})q] \quad (15)$$

The time-variant behavioral constraints are related to the corresponding structural response by means of the stress $\sigma_{i,\ell} = \sigma_{i,\ell}(t)$ and displacement $u_{j,\ell} = u_{j,\ell}(t)$ associated with each element i , nodal point j , and loading condition ℓ , as follows:

$$\begin{cases} \bar{\sigma}_{i,\ell}^- (\mathbf{x}, t) \leq \sigma_{i,\ell} (\mathbf{x}, t) \leq \bar{\sigma}_{i,\ell}^+ (\mathbf{x}, t) \\ -\sigma_{i,\ell} (\mathbf{x}, t) \leq \bar{\sigma}_i^{cr} (\mathbf{x}, t) \end{cases} \quad \bar{u}_{j,\ell}^- \leq u_{j,\ell} (\mathbf{x}, t) \leq \bar{u}_{j,\ell}^+ \quad (16)$$

where $\bar{\sigma}_{i,\ell}^- = \bar{\sigma}_{i,\ell}^-(t)$ and $\bar{\sigma}_{i,\ell}^+ = \bar{\sigma}_{i,\ell}^+(t)$ are the minimum and maximum allowable stress, respectively, $\bar{u}_{j,\ell}^-$ and $\bar{u}_{j,\ell}^+$ are prescribed displacement bounds, and $\bar{\sigma}_i^{cr} = \bar{\sigma}_i^{cr}(t)$ is the critical stability threshold. Constraints on both local and global damage may be also introduced (Azzarello *et al.* 2006). This optimization problem is solved by using a gradient-based method (Vanderplaats, 2001).

4.2 The Case of Reinforced Concrete Structures

The general approach previously presented can be easily extended to the special case of reinforced concrete structures. First, the damage evolution in concrete and steel is described by means of two damage indices $\delta_c = \delta_c(t)$ and $\delta_s = \delta_s(t)$, respectively. Second, the initial cost C_0 is computed as follows:

$$C_0 = c_c V_{c,0} + c_s V_{s,0} = c_c (V_{c,0} + \kappa V_{s,0}) = c_c V_{c,0}^* \quad (17)$$

where $V_{0,c}$, $V_{0,s}$ are the total volumes of concrete and steel, respectively, c_c , c_s are the corresponding unit costs, $\kappa = c_s/c_c$ is the unit cost ratio, and $V_{c,0}^*$ is the equivalent volume of concrete. Finally, focusing the attention on the Serviceability Limit State (SLS), the time-variant behavioral constraints on the stress state associated with each loading condition ℓ are related to the stress in both concrete fibers $\sigma_{c,i,\ell} = \sigma_{c,i,\ell}(t)$ and steel bars $\sigma_{s,i,\ell} = \sigma_{s,i,\ell}(t)$ of each element i :

$$\begin{cases} \bar{\sigma}_{c,i,\ell}^- (\mathbf{x}, t) \leq \sigma_{c,i,\ell} (\mathbf{x}, t) \leq \bar{\sigma}_{c,i,\ell}^+ (\mathbf{x}, t) \\ \bar{\sigma}_{s,i,\ell}^- (\mathbf{x}, t) \leq \sigma_{s,i,\ell} (\mathbf{x}, t) \leq \bar{\sigma}_{s,i,\ell}^+ (\mathbf{x}, t) \end{cases} \quad (18)$$

Clearly, additional constraints related to the Ultimate Limit States (ULS) may be also introduced, even though they are not considered here.

5 APPLICATIONS

5.1 Steel Truss

The proposed formulation is applied to the lifetime optimization of the truss structure shown in Figure 3.a. Three alternative loading conditions are considered: (a) $F_z=F=1$ MN and $F_y=0$; (b) $F_z=0$ and $F_y=F$; (c) $F_z=F_y=F$. It is worth noting that, in general, the response to the loading condition (c) is not given by the superposition of (a) and (b), since the system properties vary during time.

A circular cross-section with diameter d_i is assumed for each member $i=1,\dots,10$. The structural shape is defined by the coordinates (z_j, y_j) of each node $j=1,..7$. The design model is based on the following geometrical constraints: $d_1=d_4$, $d_2=d_5$, $d_3=d_6$, $d_7=d_8$, $d_9=d_{10}$, $z_1=z_4=0$; $y_1=y_4$, $(z_2, y_2)=(z_5, y_5)$, $(z_3, y_3)=(z_6, y_6)$, $y_7=0$, $z_7=L=3$ m. Therefore, the optimization problem is defined by $n=10$ design variables $\mathbf{x}=[d_1 d_2 d_3 d_8 d_9 | y_1 z_2 y_2 z_3 y_3]^T$, for which the following side constraints are assumed: $0.05 \text{ m} \leq d_i \leq 0.50 \text{ m}$; $0.10 \text{ m} \leq y_j \leq 1.00 \text{ m}$; $0.10 \text{ m} \leq z_2 \leq 2.80 \text{ m}$; $0.20 \text{ m} \leq z_3 \leq 2.90 \text{ m}$. For the behavioral constraints the initial allowable stress $\bar{\sigma}_{0,i,\ell}^+ = -\bar{\sigma}_{0,i,\ell}^- = \bar{\sigma}_0 = 160 \text{ MPa}$ is considered for each member i and loading condition ℓ .

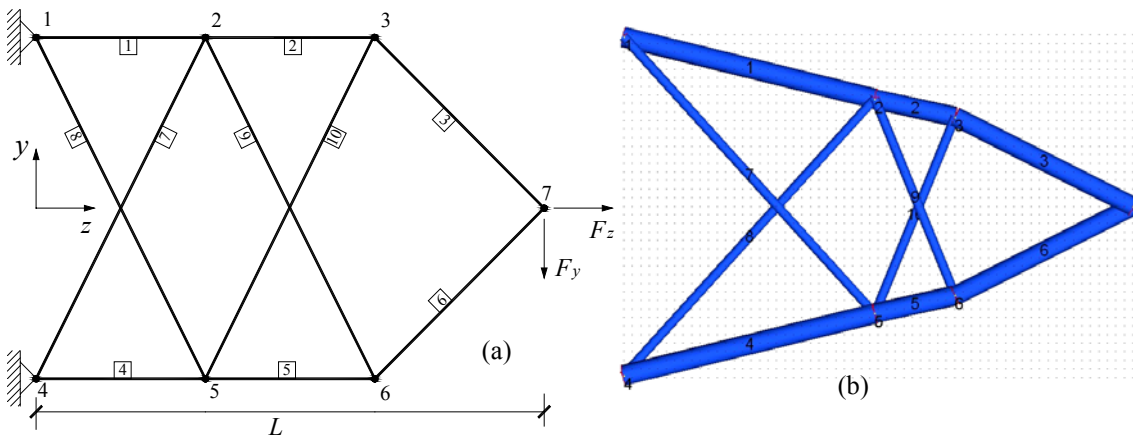


Figure 3. Truss structure. (a) Design model. (b) Optimal solution without damage ($\theta=0$).

Table 1. Truss structure. Optimal value of the design variables without damage ($\theta=0$).

d_1 [mm]	d_2 [mm]	d_3 [mm]	d_8 [mm]	d_9 [mm]	y_1 [mm]	z_2 [mm]	y_2 [mm]	z_3 [mm]	y_3 [mm]
120.5	116.5	115.7	57.7	57.8	988.9	1478.6	627.2	1952.2	522.2

In case damage is not considered, the optimal solution described in Figure 3.b and Table 1 is achieved. This solution is compared with the structures shown in Figure 4, obtained by solving the lifetime optimization problem with reference to a service lifetime $T_S=100$ years and a damage rate $\theta=T_S/T_\delta=10$, with $\sigma_{cr}=0$ and $\alpha=1$, for different design monitoring periods $T_D=\{1, 50 \text{ years}\}$ and discount rates $\nu=\{0, 3\%\}$. This comparison, together with the results obtained for other values of the parameters ν and T_D (Azzarello, 2005), leads to the following general conclusions:

- The optimal design solution \mathbf{x}^* strongly depends on both the parameters T_D and ν . In particular, both such parameters influence the optimal dimension of the cross-sections, while the ν -values tend to significantly affect the optimal location of the nodal points only.
 - The initial cost χ_0^* strongly increases with T_D , and does not significantly depend on ν .
 - The maintenance cost χ_m^* decreases with both T_D and ν .
 - The total cost χ^* increases with T_D , and decreases with ν . This tendency highlights the expected higher cost-effectiveness of a maintenance planning with frequent interventions over more conventional maintenance-free design strategies.
 - The lifetime damage index δ increases with T_D , and does not significantly depend on ν .
 - Despite no constraints are imposed on the nodal displacements, the maximum deflection does not significantly depend on T_D and ν . In particular, the maximum vertical displacement of node 7 approximately equals the optimal value associated with the undamaged scenario ($u_{y7}=5.1 \text{ mm}$).
- Similar results are obtained by varying the parameter θ in the range $0.50 \div 20$ (Azzarello, 2005). Clearly, higher θ -values lead to more ponderous optimal structures and, consequently, to higher values of the optimal cost components.

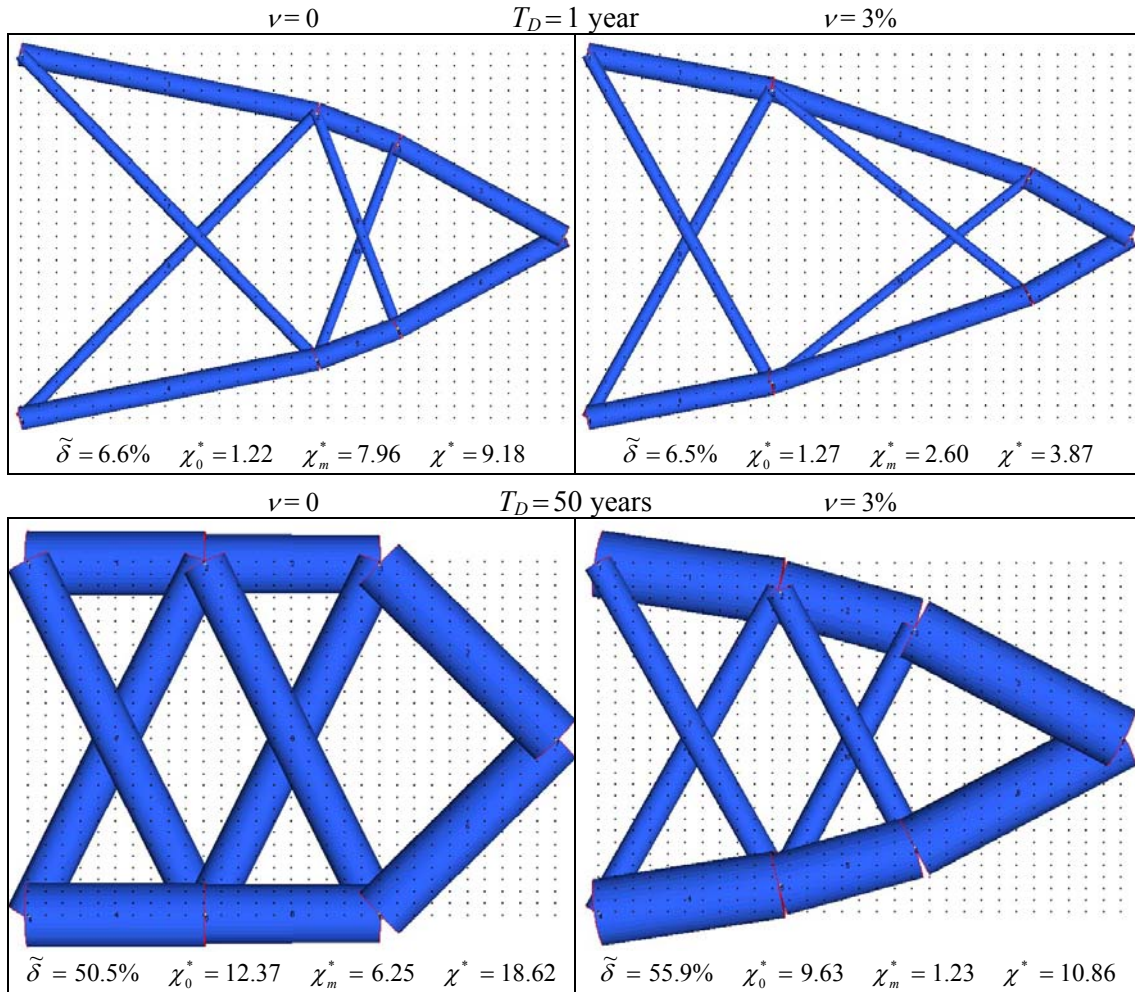


Figure 4. Truss structure. Optimal solutions in a design scenario with damage ($\theta = T_S/T_D = 10$, $T_S = 100$ years) for different values of the design monitoring period $T_D = \{1, 50\}$ years and of the discount rate $\nu = \{0, 3\}\%$.

5.2 Reinforced Concrete Frame

The previous formulation is applied to the lifetime optimization of the reinforced concrete frame shown in Figure 5.a. A rectangular cross-section is assumed for both beam and columns. With reference to Figures 5.a and 5.b, the optimization problem is defined by $n=9$ design variables $\mathbf{x} = [b \ h_1 \ h_2 \ | \ A_1 \ A_2 \ A_2' \ A_3 \ A_3' \ | \ d]^T$, for which the following side constraints are assumed: $b \geq 300$ mm; $1 \leq h_j/b \leq 2$, $i=1,2$; $A_i \geq 2\emptyset 12$ and $A_j' \geq 2\emptyset 12$, $i=1,2,3$; $0.10 \leq d/L \leq 0.50$.

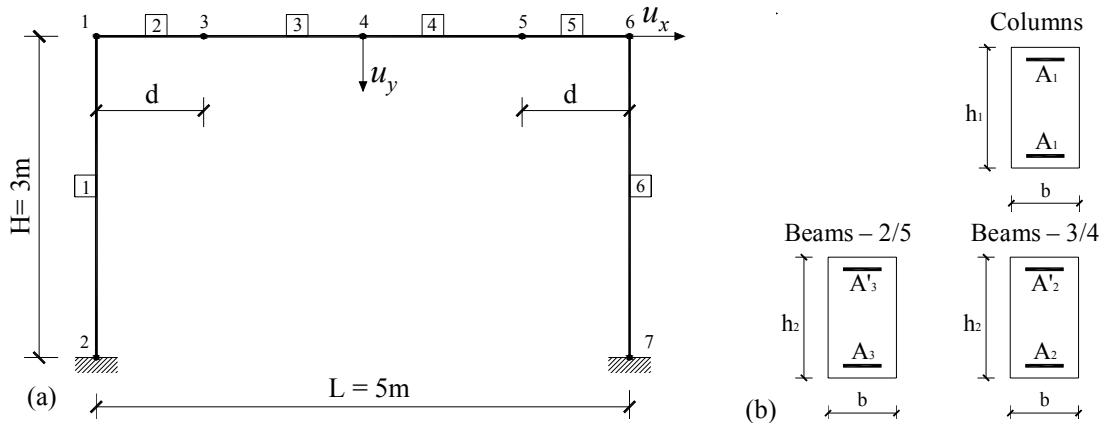


Figure 5. Reinforced concrete frame. (a) Geometrical dimensions and structural model. (b) Cross-sections of beam and columns.

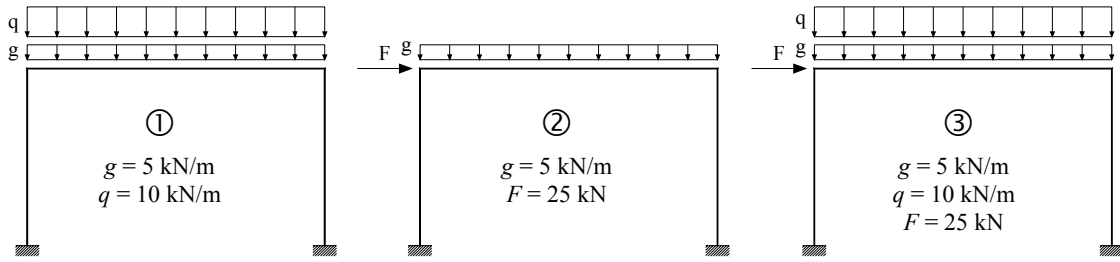


Figure 6. Reinforced concrete frame. Loading conditions.

Table 2. Reinforced concrete frame. Damage scenarios ($\theta = T_S/T_\delta = 10$, $T_S = 100$ years).

Damage Scenario	Left Column	Beam	Right Column
A	-	-	-
B	Damage	Damage	Damage
C	-	Damage	-
D	Damage	-	Damage
E	Damage	Damage	-
F	-	Damage	Damage

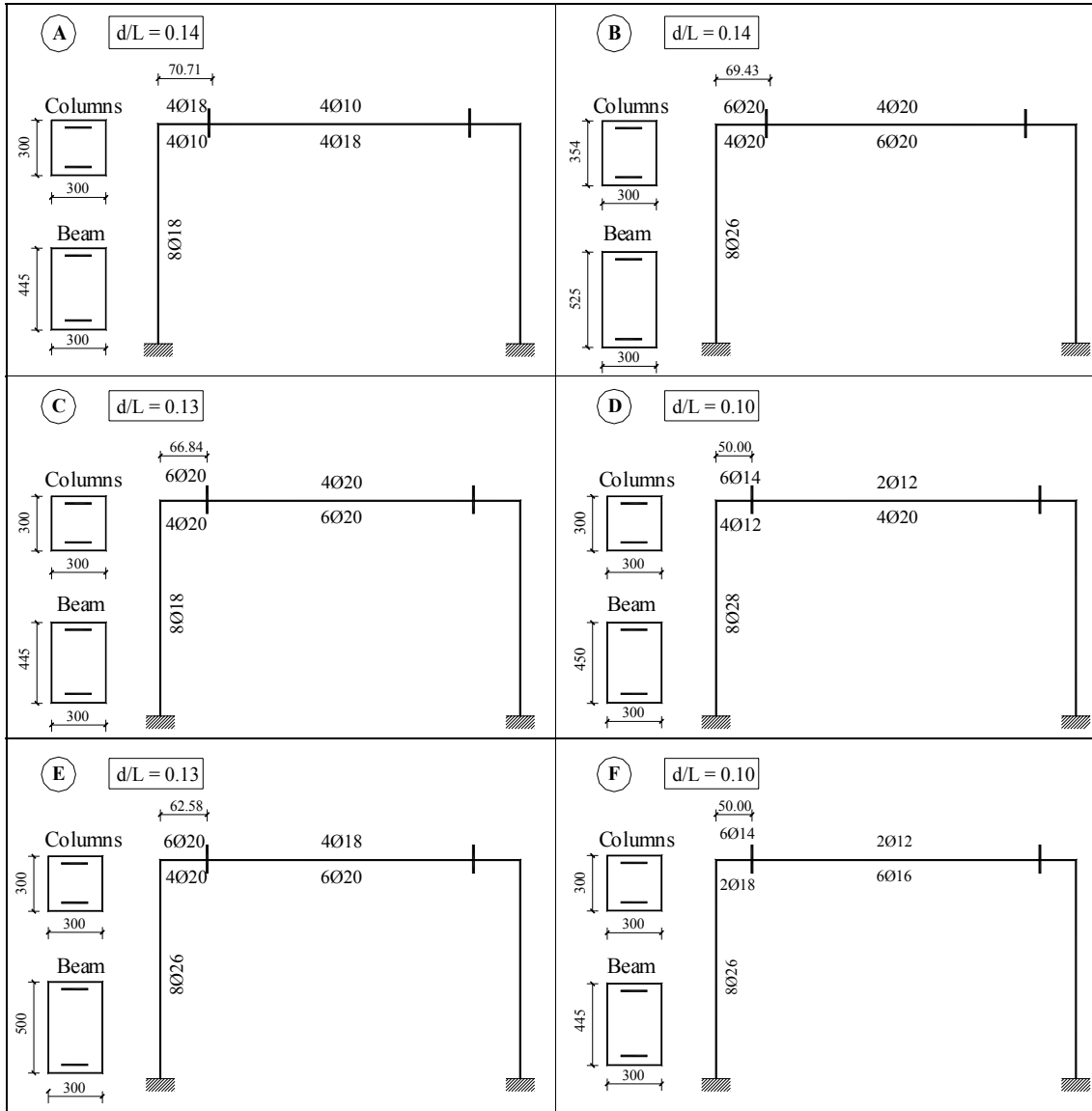


Figure 7. Reinforced concrete frame. Optimal design solutions associated with the damage scenarios described in Table 2 ($\theta = T_S/T_\delta = 10$, $T_S = 100$ years, $T_D = 10$ years, $\kappa = 20$, $\nu = 3\%$).

The three alternative loading conditions shown in Figure 6 are considered. Since the SLS is investigated, the time-variant structural response in terms of nodal displacements and internal stress resultants are evaluated by assuming a linear elastic behavior. Based on the general criteria for concrete design, the corresponding stress in the materials are computed at the cross-sectional level by assuming for concrete a linear elastic behavior in compression with $E_c=30$ GPa and no strength in tension ($\bar{\sigma}_c^+ = 0$), and for steel a linear elastic behavior in both tension and compression with $E_s=15E_c$.

The same behavioral constraints are considered for each loading condition. The stress in the materials is verified in each member by assuming the initial allowable stress $-\bar{\sigma}_{c,0}^- = 15$ MPa for concrete, and $\bar{\sigma}_{s,0}^+ = -\bar{\sigma}_{s,0}^- = 180$ MPa for steel. The displacement constraints $u_x \leq 20$ mm and $u_y \leq 10$ mm over the whole service lifetime are also considered (Figure 5.a).

The lifetime performances are defined by the following parameters: service lifetime $T_S=100$ years, design period $T_D=10$ years, unit cost ratio $\kappa=20$, discount rate $\nu=3\%$, and damage rate $\theta=T_S/T_D=10$, with $\sigma_c=0$ and $\alpha=1$ for both concrete and steel. Moreover, to investigate the role of the spatial distribution of deterioration, the damage scenarios described in Table 2 are considered.

The direct comparison of the optimal solutions shown in Figure 7 shows that the optimal dimensions of the cross-sections, as well as the optimal amount and distribution of reinforcement, strongly depend on the prescribed damage scenario. In particular, it is worth noting that damage leads to a more ponderous design not only for members affected by deterioration. In fact, due to redundancy, damage induces a time-variant redistribution process in which the internal stress resultants tend to progressively move towards the undamaged members.

6 CONCLUSIONS

A new conceptual approach to the minimum lifetime cost of deteriorating structures under multiple loading conditions has been presented. This approach allowed to overcome the inconsistencies involved in the classical formulation of the optimum design problem, where the time evolution of the structural performance induced by the progressive deterioration of the system properties is not adequately considered. In fact, in the proposed formulation, the structural damage is accounted for by means of a proper material degradation law of the mechanical properties, and the design constraints of the optimization problem are related to the corresponding time-variant structural performance over the whole expected service lifetime of the construction. In addition, the objective function is formulated by accounting for both the initial cost of the structure and the costs of possible maintenance interventions, that are properly discounted over time and assumed to be proportional to the actual level of structural damage.

The lifetime optimization of a steel truss and of a reinforced concrete frame proved the effectiveness of the proposed approach by highlighting that the optimal solution strongly depends on both the time-variant structural performance and the maintenance planning.

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