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Lifetime Nonlinear Analysis of Concrete Structures under Uncertainty

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ABSTRACT: In concrete structures the relative importance of the uncertainty effects associated with each random variable may significantly vary during time due to the various sources of structural damage which may strongly modify the structural reliability. This paper presents the results of a preliminary investigation aimed to highlight the time evolution of the uncertainty effects associated with the different parameters which define the structural performance. The study is developed by using a novel methodology for time-variant reliability analysis of concrete structures subjected to diffusive attacks from external aggressive agents. Based on this procedure, a probabilistic time-variant nonlinear analysis of a concrete box-girder cross-section undergoing diffusion is carried out by Monte Carlo simulation and the corresponding time evolution of the structural performance is investigated with reference to suitable indicators of the nonlinear behavior. The effects of each random variable on such indicators are finally quantified and compared by means of a suitable time-dependent sensitivity factor.

1 INTRODUCTION

Reliability based analysis and design procedures must account for the uncertainties which affects the structural performance. However, for concrete structures the number of uncertain parameters involved in design is usually large, and probabilistic evaluations may become very complex. This aspect is particularly emphasized when full nonlinear analyses are required to investigate the structural performance. For this reason, in design practice the complexity of the reliability problem is reduced by means of simplified probabilistic methods in which only a few uncertain parameters are considered as random variables, typically only those which mainly affect the structural response. In concrete design such variables are generally associated with the material properties, i.e. concrete and steel strengths, while for the other mechanical and geometrical parameters the deterministic nominal values are assumed.

This design approach is calibrated by the codes to be on the safe side and is proven to be effective for practical purposes. However, such calibration mainly refers to undamaged structures and does not account for the effects of uncertainty associated with the unavoidable sources of structural damage which during time may strongly modify the reliability level. In fact, for concrete structures immersed in aggressive environments the structural performance must be considered as time-dependent, mainly because of the progressive deterioration of the mechanical properties of materials which makes the structural system less able to withstand the applied actions. In this context, a design procedure aimed to achieve the required level of structural performance not only at the initial time, but over the whole expected service life of the structure, must consider that the relative importance of the uncertainty effects associated with each design parameter may vary significantly during time. To this aim, the role played by both the deterioration process and the corresponding evolution of the structural behavior on this variation needs to be investigated and clarified. In this way, it would be possible to calibrate the design procedures with respect to the random variables which actually affect the lifetime structural response.

This paper presents the results of a preliminary investigation aimed to highlight the time evolution of the uncertainty effects associated with the different parameters which define the structural performance. The study is developed by using a novel methodology for time-variant reliability analysis of concrete structures subjected to diffusive attacks from external aggressive agents, recently proposed by the authors (Biondini et al. 2004*a*, 2004*b*, 2006). The diffusion process is modeled by using a special class of evolutionary algorithms, called cellular automata, and the mechanical damage coupled to diffusion is evaluated by introducing proper material degradation laws. Since the rate of mass diffusion usually depends on the stress state, the interaction between the diffusion process and the mechanical behavior of the damaged structure is also taken into account by a proper modeling of the stochastic effects in the mass transfer.

Based on this procedure, a probabilistic time-variant nonlinear analysis of a concrete box-girder cross-section undergoing diffusion is carried out by Monte Carlo simulation and the corresponding time evolution of the structural performance is investigated with reference to suitable indicators on the nonlinear behavior. The effects of the uncertainty associated with each random variable on such indicators are finally quantified and compared by means of a time-dependent sensitivity factor based on a linear regression of the simulation results.

2 TIME-VARIANT PERFORMANCE IN AGGRESSIVE ENVIRONMENTS

2.1 *Case Studied*

The present study refers to the arch bridge over the Corace river in Italy (Galli and Franciosi 1955). The time-variant collapse reliability of the whole bridge structure under prescribed damaging scenarios has been analyzed in a previous work (Biondini and Frangopol 2006). In this study, the attention is focused on the nonlinear behavior of the two-cellular cross-section which characterizes a segment of the box-girder deck.

The nominal geometrical dimensions of the cross-section are: width = 6.00 m; height = 2.00 m; web thickness = 0.20 m; top slab thickness = 0.18 m; bottom slab thickness = 0.16 m. The cross-section is reinforced at the top with 48 bars having nominal diameter ∅=28 mm, and 130 bars with ∅=8 mm; at the bottom is reinforced with 21 bars having ∅=28 mm. Figure 1.a shows the discretization of the structural model.

The nonlinear bending behavior of the cross-section is defined by assigning the constitutive laws of the materials. For concrete, the stress-strain diagram is described by the Saenz's law in compression and by an elastic perfectly plastic model in tension, with the following nominal parameters: compression strength f_c=–30 MPa; tension strength $f_{ct}=0.25|f_c|^{2/3}$; initial modulus $E_{c0}=9500|f_c|^{1/3}$; peak strain in compression ε_{c0}=–0.20%; strain limit in compression ε_{cu}=–0.35%; strain limit in tension $\varepsilon_{ctu}=2f_{ct}/E_{c0}$. For steel, the stress-strain diagram is described by an elastic perfectly plastic model in both tension and compression, with the following nominal parameters: yielding strength f_{sy}=300 MPa; elastic modulus E_s=206 GPa; strain limit ε_{su}=1.00%.

2.2 *Diffusion Process*

The cross-section is assumed to be subjected to a diffusive attack from an environmental aggressive agent, which is considered to be located with concentration $C(t)=C_0$ along the free edges, as shown in Figure 1.b. The kinetic diffusion process is described according to the Fick's laws and is effectively simulated by using a special class of evolutionary algorithms called cellular automata. In particular, it can be shown that the Fick's laws in two-dimensions can be simply reproduced by adopting the following evolutionary rule (Biondini et al. 2004*a*):

$$C_i^{k+1} = \phi_0 C_i^k + \frac{1-\phi_0}{4}\sum_{j=1}^{2}(C_{i-1,j}^k + C_{i+1,j}^k) \tag{1}$$

where the discrete variable $C_i^k = C(\mathbf{x}_i, t_k)$ represents the concentration of the component in the cell i at time t_k, and ϕ_0 is a suitable evolutionary coefficient. The deterministic value $\phi_0 = 1/2$ usually leads to a good accuracy of the automaton. Clearly, to regulate the process according to a given diffusivity coefficient D, a proper discretization in space and time should be chosen in such a way that the grid dimension Δx and the time step Δt satisfy the following relationship:

$$D = \frac{1-\phi_0}{4}\frac{\Delta x^2}{\Delta t} \tag{2}$$

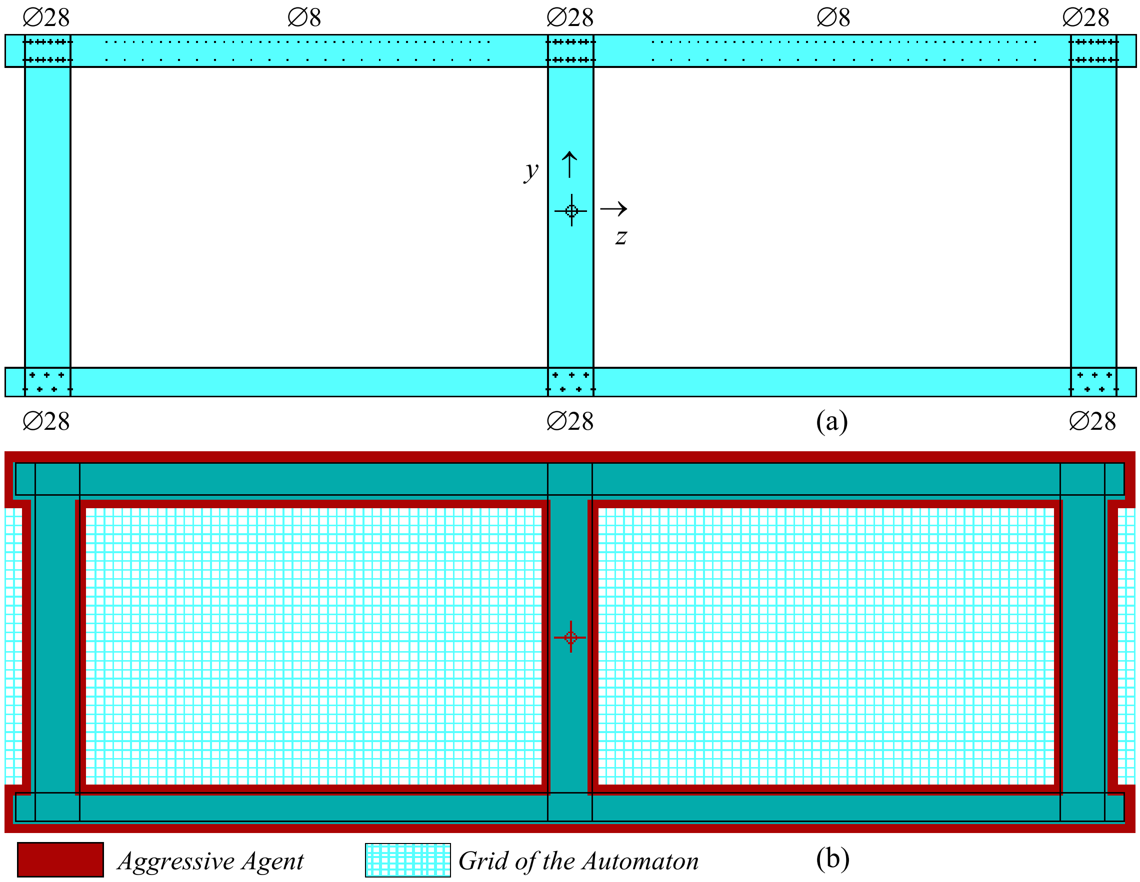

Figure 1. Cross-section of a box-girder bridge (main nominal dimensions 6.00×2.00 m). (a) Discretization of the structural model. (b) Grid of the cellular automaton and location of the aggressive agent.

The cellular automaton associated to the considered cross-section is shown in Figure 1.b. With reference to a nominal diffusivity coefficient $D = 10^{-11}$ m^2/sec, the automaton is defined by a grid dimension $\Delta x = 50.2$ mm and a time step $\Delta t = 1$ year. The diffusion process for the nominal scenario is described by the maps of concentration $C(\mathbf{x},t)/C_0$ shown in Figure 2.

2.3 *Structural Damage and Nonlinear Analysis*

Structural damage is modeled by introducing a degradation law of the effective resistant area for both the concrete matrix and the steel bars. This degradation is achieved by means of proper dimensionless *damage indices* δ_c and δ_s for steel and concrete, respectively, which provide a direct measure of the damage level of the materials within the range [0; 1]. The damage indices $\delta_c = \delta_c(\mathbf{x}, t)$ and $\delta_s = \delta_s(\mathbf{x}, t)$ at point $\mathbf{x} = (y, z)$ and time t are correlated to the diffusion process by assuming, for both materials, a linear relationship between the rate of damage and the mass concentration $C = C(\mathbf{x}, t)$ of the aggressive agent (Biondini et al. 2004*a*):

$$\frac{\partial \delta_c(\mathbf{x},t)}{\partial t} = \frac{C(\mathbf{x},t)}{C_c \Delta t_c} \qquad \frac{\partial \delta_s(\mathbf{x},t)}{\partial t} = \frac{C(\mathbf{x},t)}{C_s \Delta t_s} \tag{3}$$

where C_c and C_s represent the values of constant concentration $C(\mathbf{x},t)$ which lead to a complete damage of the materials after the time periods Δt_c and Δt_s, respectively. In addition, the initial conditions $\delta_c(\mathbf{x},t_0)=\delta_s(\mathbf{x},t_0)=0$ with $t_0 = \max\{t \mid C(\mathbf{x},t) \le C_{cr}\}$ are assumed, where C_{cr} is a critical threshold of concentration. Since the rate of mass diffusion depends on the stress state, the interaction between diffusion process and mechanical behavior of the damaged structure is also taken into account by a proper modeling of the random variable ϕ_0, which describes the stochastic effects in the mass transfer (Biondini et al. 2004*a*).

In the present study, a very severe damage scenario with nominal values $C_{cr} = 0$, $C_c = C_s = C_0$, $\Delta t_c = 25$ years and $\Delta t_s = 50$ years, is considered. The mechanical damage induced by diffusion for the nominal scenario can be evaluated from Figure 3, which shows the time evolution of the bending moment M versus curvature χ diagrams.

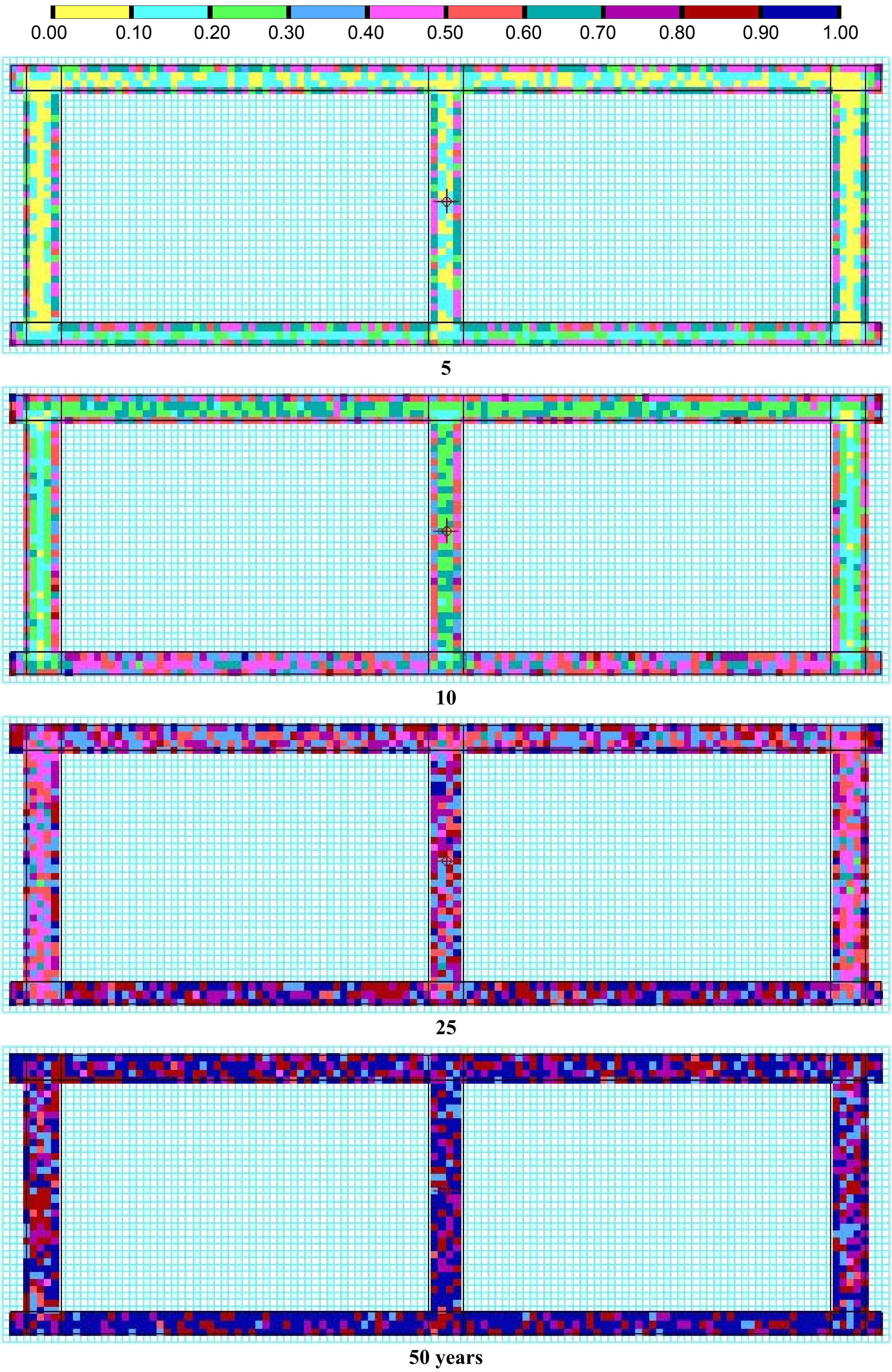

Figure 2. Maps of concentration $C(\mathrm{x},t)/C_0$ of the aggressive agent after 5, 10, 25, and 50 years from the initial time of diffusion penetration (nominal scenario).

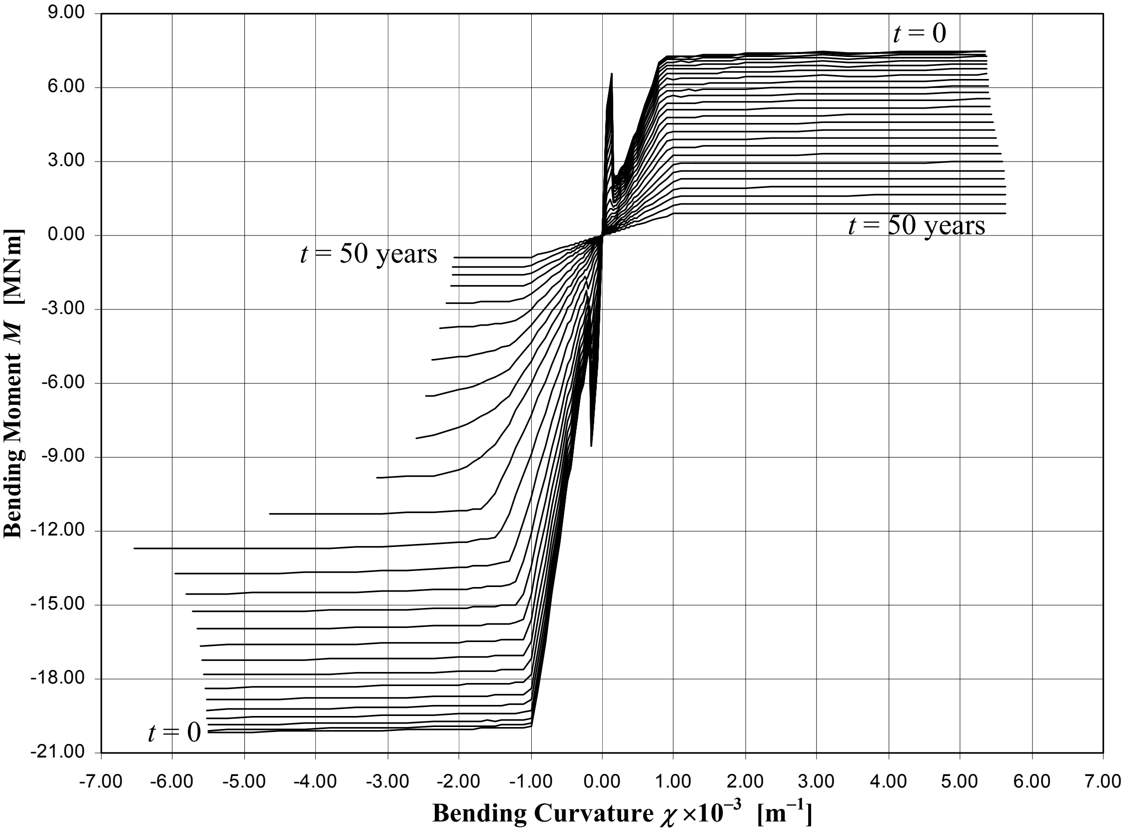

Figure 3. Structural response during the first 50 years of lifetime (Δt=2 years) in terms of bending moment M versus curvature χ diagrams (nominal scenario).

3 THE ROLE OF UNCERTAINTY IN LIFETIME NONLINEAR ANALYSIS

3.1 *Probabilistic Analysis*

The probabilistic model assumes as random variables the material strengths f_c and f_{sy}, the coordinates (y_p, z_p) of the nodal points p=1,2,… which define the two-dimensional model of the concrete cross-section, the coordinates (y_m, z_m) and the diameter $\varnothing_m$ of the steel bars m=1,2,…, the diffusion coefficient D, and the damage rates $q_c=(C_c\Delta t_c)^{-1}$ and $q_s=(C_s\Delta t_s)^{-1}$. These variables are assumed to have the probabilistic distribution with the mean μ and standard deviation σ values listed in Table 1 (Biondini *et al.* 2004*b*, 2006).

With reference to the bending moment versus curvature diagram shown in Figure 4, the cracking moment M_{cr} and the resistant moment M_r, as well as the curvature ductility $\varphi=\chi_u/\chi_y$ given by the ratio of curvatures at ultimate and yielding, respectively, are assumed as suitable indicators of structural performance.

Table 1. Probability distributions and their parameters.

Random Variable (t=t_0)	Distribution Type	μ	σ
Concrete strength, f_c	Lognormal	$f_{c,nom}$	5 MPa
Steel strength, f_{sy}	Lognormal	$f_{sy,nom}$	30 MPa
Coordinates of the nodal points, (y_p, z_p)	Normal	$(y_p, z_p)_{nom}$	5 mm
Coordinates of the steel bars, (y_m, z_m)	Normal	$(y_m, z_m)_{nom}$	5 mm
Diameter of the steel bars, $\varnothing_m$	Normal (*)	$\varnothing_{m,nom}$	$0.10\varnothing_{m,nom}$
Diffusivity coefficient, D	Normal (*)	D_{nom}	$0.10\ D_{nom}$
Concrete damage rate, q_c	Normal (*)	$q_{c,nom}$	$0.30\ q_{c,nom}$
Steel damage rate, q_s	Normal (*)	$q_{s,nom}$	$0.30\ q_{s,nom}$

(*) Truncated distributions with non negative outcomes are adopted in the simulation process.

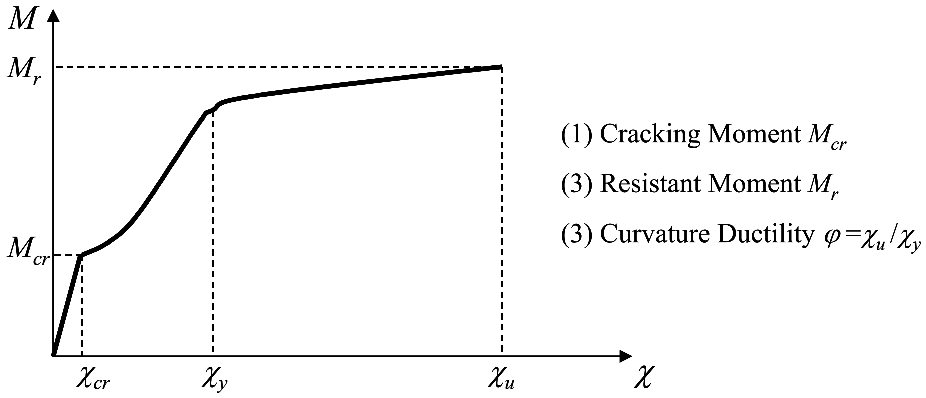

Figure 4. Structural performance indicators of the bending cross-sectional response.

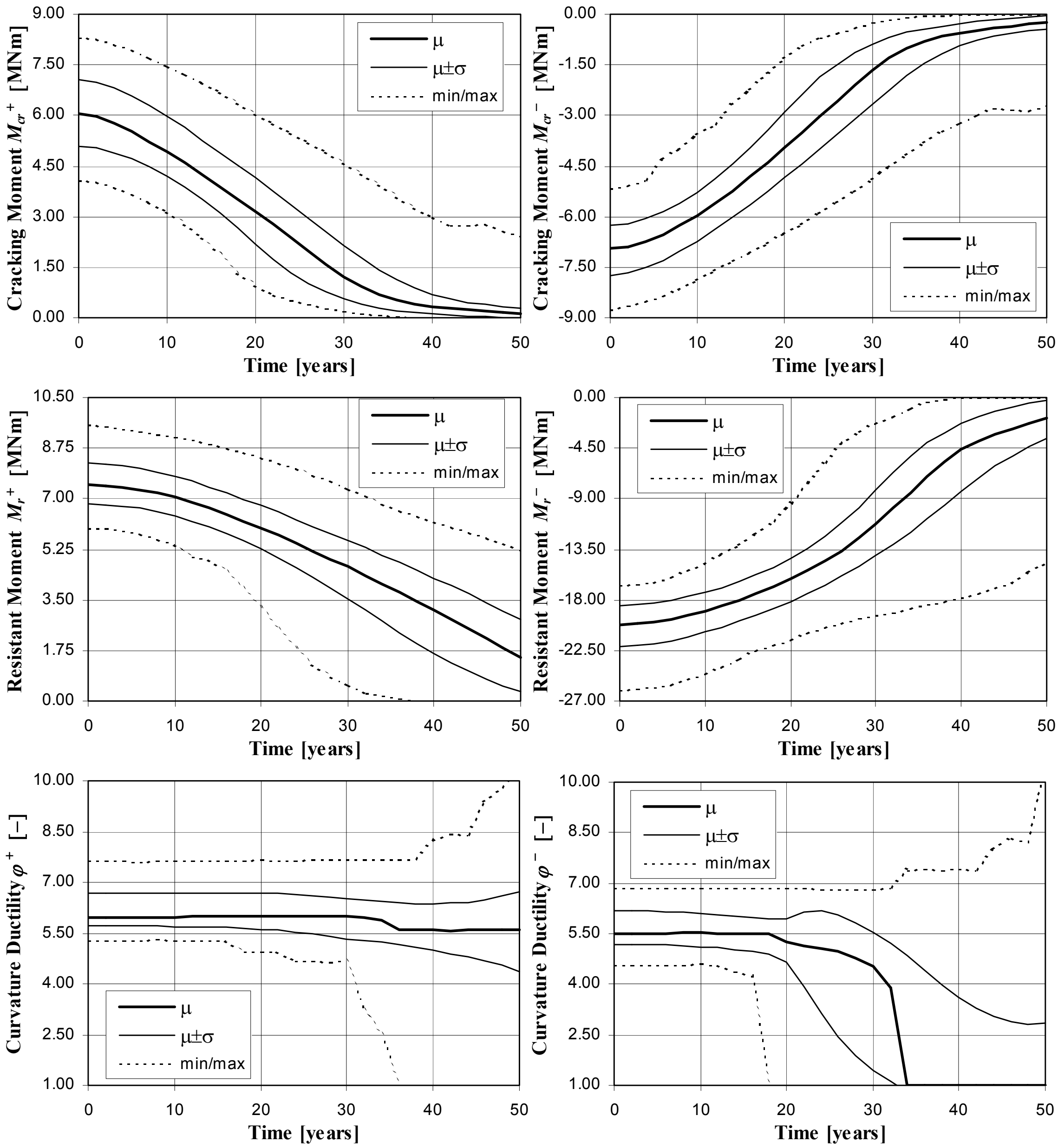

Figure 5. Time evolution of the structural performance indicators during the first 50 years of service life: mean μ (thick line), standard deviation σ from the mean μ (thin lines), minimum and maximum values (dotted lines).

With reference to a sample of 5000 simulations, Figure 5 shows the time evolution of the statistical parameters (mean value μ, standard deviation σ, minimum and maximum values) of the performance indicators for both positive and negative bending responses during the first 50 years of service life. When directly compared with the random variability of the structural demand, the results of the probabilistic simulation allow to assess the time-variant reliability of the cross-section or, conversely, to assess the corresponding remaining service life which can be assured under prescribed reliability levels without maintenance.

3.2 *Regression Analysis and Sensitivity Factors*

The results of the simulation process highlight that the deterioration of structural performance tends to be associated with a significant increase of the spreading effects of uncertainty over time (Figure 5). This tendency is particularly emphasized for curvature ductility. However, as already pointed out, the relative importance of the uncertainty effects associated with each one of the random variables a_i listed in Table 1 may significantly vary during time.

In order to investigate the effects of the uncertainty associated with each parameter x_i on each performance indicator y_j, the following standard normal variates are introduced:

$$\xi_i(t) = \frac{x_i(t) - \mu_{x,i}(t)}{\sigma_{x,i}(t)} \qquad \eta_j(t) = \frac{y_j(t) - \mu_{y,j}(t)}{\sigma_{y,j}(t)} \tag{4}$$

where μ_x, σ_x, and μ_y, σ_y, are the time-variant mean value and standard deviation of the random variables x and y, respectively. Based on these variates, a set of time-variant least squares linear regression is performed on the data samples obtained from the simulation in the following form:

$$\eta_j(t) = \alpha_{ij}(t)\xi_i(t) + \beta_{ij}(t) \tag{5}$$

In this way, the regression coefficients α_{ij} are assumed as a time-dependent measure of the sensitivity of the dependent variable y_j with respect to the independent variable x_i.

The results of this analysis are shown in Figure 6 where, for the sake of synthesis, the sensitivity factors associated with the coordinates of the nodal points of the cross-sectional model, and with both the coordinates and areas of the steel bars, are computed with reference to the mean values of the corresponding standard variates over the whole cross-section and denoted with y_c, y_s, and A_s, respectively. With reference to the diagrams of Figure 6 the following remarks can be made:

- The cracking moments in the undamaged scenario mainly depend on the concrete strength. Such dependency quickly decreases during time, and after about 20 years the damage rate of concrete becomes the more important parameter. However, this contribution tends to progressively disappear. At the end of service life an important role is played by the damage rate of steel and, for the positive cracking moment, by the geometrical dimensions of the cross-section.
- The resistant moments in the undamaged scenario mainly depend on the steel strength. Such dependency quickly decreases during time and become negligible after about 25 years. At this point, for positive resistant moment the damage rate of steel becomes the more important parameter for the whole remaining service life. On the contrary, for negative resistant moment a clear dependency no longer holds until the last years of service life, when the damage rate of steel begins to give a significant contribution.
- The curvature ductility in the undamaged scenario mainly depend on the steel strength. For positive bending behavior such dependency is maintained along the whole service life. On the contrary, for negative bending behavior more complex correlations emerge after about 15-20 years, when the role played by the location of the steel bars and the damage rate of concrete becomes more important. However, these contributions tend to disappear at the end of service life, when a significant contribution is given by the area of the steel bars.

4 CONCLUSIONS

The results of this investigation proved that for concrete structures in aggressive environments the relative importance of the uncertainty effects associated with each design parameter may significantly vary during time. In particular, the classical view in which the main role in concrete design is played by the uncertainty associated with the material strengths needs to be reviewed to account for the effects of the unavoidable sources of structural damage which during time may strongly modify the structural performance and the corresponding reliability level.

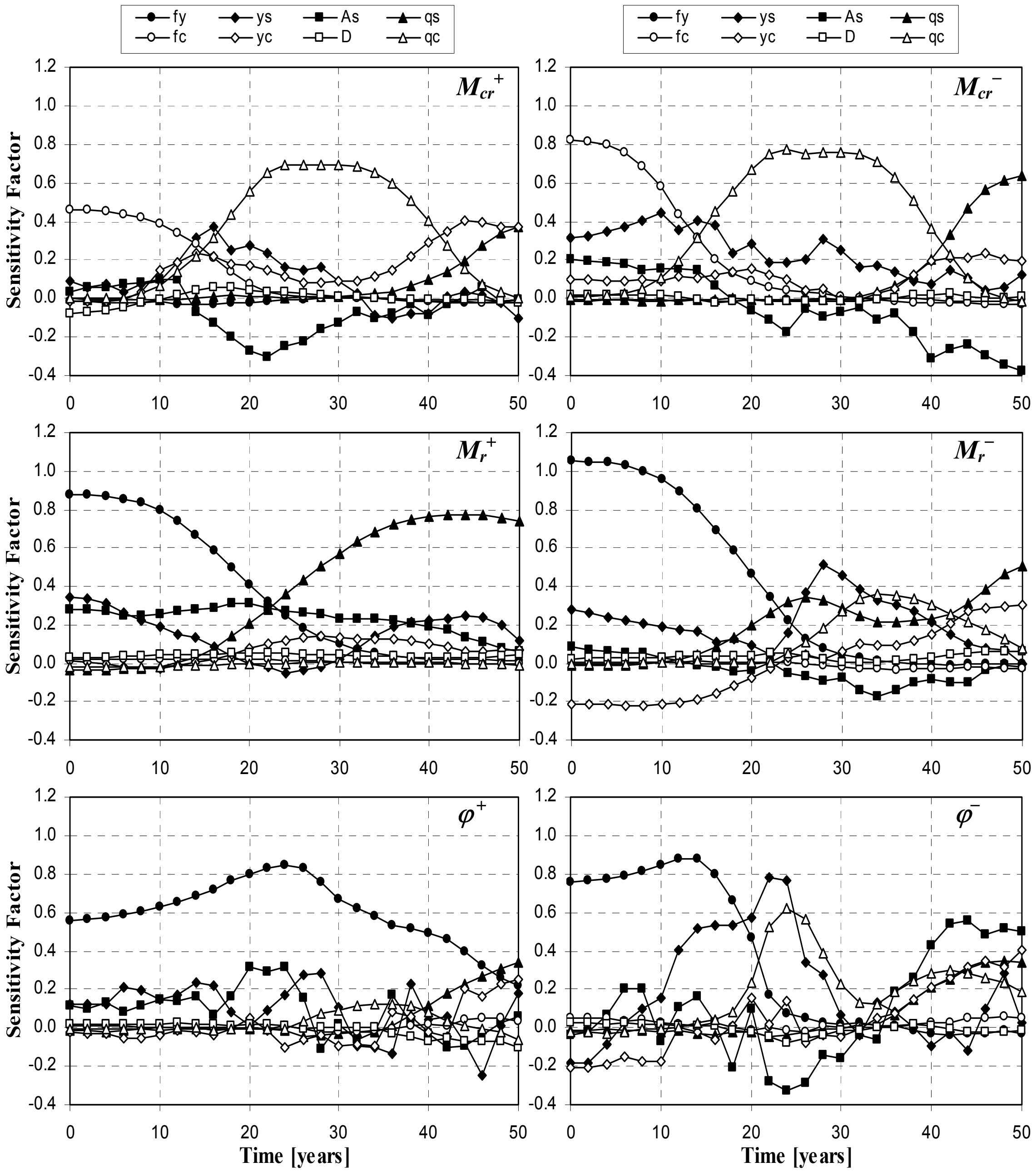

Figure 6. Sensitivity factors of the structural performance indicators.

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