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Markovian Modeling for Lifetime Prediction and Maintenance Planning of Deteriorating Structures

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ABSTRACT: This paper presents a probabilistic model for the service life assessment of deteriorating structures based on a few observed data. Damage evolution is modeled as a semi-Markov process and its formulation allows to account for eventual improvements of the structural performance. In this way, the model can be also used to plan maintenance and/or rehabilitation interventions. The proposed procedure is applied to the service life prediction of a bridge pier and to the selection of a suitable preventive maintenance scenario.

1 INTRODUCTION

Structural systems in aggressive environments are subjected during time to a deterioration of their mechanical properties. This process makes the system less able to withstand the applied actions and, as a consequence, tends to decrease its structural reliability. Therefore, when the reliability level reaches a prescribed target lower threshold, the system is no longer able to satisfy the requirements for which it was conceived and, unless some maintenance or rehabilitation interventions are performed, the end of the design service life occurs.

In order to assess the service life of a structural system interacting with an aggressive environment, and to eventually plan a process of essential and/or preventive maintenance, some basic information on the actual development of the deterioration process, as well as on the uncertainty involved in its modeling, is clearly required. A valuable set of useful data can be often found from monitoring activities, but, as well known, due to economic reasons such activities tend to be very limited in space and time and only a few observed data are usually available.

Based on such considerations, the aim of this paper is to present a probabilistic model able to assess the service life of deteriorating structures by using only a few observed data (Biondini *et al*. 2004*a*, Garavaglia *et al*. 2004). Damage evolution is modeled as a semi-Markov process with one step of memory (Guagenti *et al*. 1988) and its formulation allows to account for eventual improvements of the structural performance. In this way, it can be also used as a tool to plan maintenance and/or rehabilitation interventions. The proposed procedure is applied to the prediction of the service life of a bridge pier and to the selection of a suitable preventive maintenance scenario.

2 RELIABILITY OF DETERIORATING STRUCTURAL SYSTEMS

The deterioration of the mechanical properties of structural systems under environmental attacks may be dealt as a reliability problem where every loss of performance greater than prescribed threshold values is considered as a "*failure*" (Sarja, 1996). Therefore, when a failure is reached the system passes from the current service state to another service state characterized by a lower level of performance. On the other hand, structural performance can also be improved by maintenance and/or rehabilitation interventions. In this case the system may move from the current service state to another service state characterized by a higher level of performance. However, in both cases the *failure process* may be defined as a *transition process* through different service states due to a number of environmental attacks and/or maintenance actions during time (Guagenti *et al.,* 1988). Clearly, since the problem is affected by several sources of uncertainty, a description of the deteriorating process and of the corresponding change in reliability can be developed only in probabilistic terms.

Based on such a vision, in this paper the evolution of the structural performance over time will be considered as a stochastic process of the random variable τ_i, where τ_i is the "*lifetime*" of the system spent in a given *service state i* (Binda and Molina 1990).

From this point of view, the time-evolution of the probability of failure P_f can be defined as follows:

$$P_f(t,t_0) = \Pr\{\tau_i \le t\} = F_{\tau_i}(t,t_0) \quad (1)$$

where t_0 represents the age of the system when it enters the state i, and $F_{\tau i}$ is the cumulative distribution function of the random variable τ_i.

The stochastic process representing the progressive failure of the deteriorating system is here modeled as a semi-Markov process (s-MP). Such kind of modeling allows to distinguish among different states of the system, each of them associated to a different waiting time, and to take in account the age t_0 of the system when a new failure process starts (Garavaglia *et al.* 2004).

3 SEMI-MARKOVIAN MODELING

A s-MP is a one-step memory process describing the behavior of a dynamic system that changes its state at every transition. A s-MP appears to be a suitable model for representing the failure process of a structural system when the following quantities are known (Howard, 1971):

a. Initial conditions, defined by the state occupied by the system at the initial time $t = 0$, and the time t_0 spent in the initial state (Fig. 1).
b. Probability density function (p.d.f.) $f_{ik}(t)$ of the waiting time τ_{ik}, i.e., the time spent in state i if the next state is k:

$$f_{ik}(t) = \Pr\{t < \tau_{ik} \le t + dt\} \quad (2)$$

c. Transition probability p_{ik}, defined as:

$$p_{ik} = \Pr\{\text{next state } k, \text{present state } i\} \quad (3)$$

Clearly, a s-MP can be characterized by different transition probabilities and different waiting times for each transition.

As said before, the time τ_i is the random variable representing the waiting time in the state i (i.e. the lifetime in the state i) between two subsequent transitions, whatever is the next destination, and its p.d.f. is defined as follows:

$$f_{\tau_i}(t) = \sum_k f_{ik}(t)\, p_{ik} \quad (4)$$

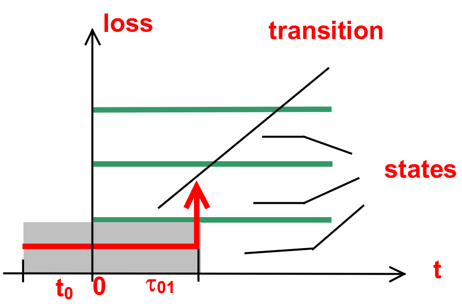


Figure 1. Schematic representation of the transition process.

The probability $\Pi_{ij}(t,t_0)$ that the system will occupy the state j at the time t if it was in the state i at the time $t = 0$, can be computed as follows:

$$\prod_{ij}(t,t_0) = \delta_{ij}\, {}_f\Im_{\tau_i}(t,t_0) + \sum_k {}_f p_{ik}(t_0) \int_0^t {}_f f_{ik}(u,t_0) \prod_{kj}(t-u)\,du \quad (5)$$

where δ_{ij} is the Kronecker operator, ${}_fF_{\tau i}(t)$ and ${}_f\Im_{\tau i}(t)$ are, respectively, the cumulative distribution and the survival function related to the first waiting time in the initial state, and:

$$\prod_{kj}(t) = \delta_{kj}\, {}_f\Im_{\tau_k}(t) + \sum_r {}_f p_{kr} \int_0^t {}_f f_{kr}(u) \prod_{rj}(t-u)\,du \quad (6)$$

The two equations above are convolution-integrals and can be effectively computed by using the Laplace transform.

Equation (5) represents the evolution of the system and allows for the prediction of the lifetime τ_i. Thus, in the semi-Markov hypothesis, the failure prediction only depends on the transition probabilities p_{ik}, on the waiting time p.d.f. $f_{ik}(t)$, and on the initial conditions.

The problem of a suitable choice for the p.d.f. $f_{ik}(t)$ arises. Obviously the choice of a suitable distribution has to be connected to the physical aspect of the phenomenon to be modeled and to the characteristics of the distribution function, especially in its tail where often no experimental data are available. In particular, the latter aspect can be investigated by analyzing the behavior of the failure rate function:

$$\phi_{ik}(t)dt = P\left\{t < \tau_{ik} \le t + dt \,\middle|\, \tau_{ik} > t\right\} \quad (7)$$

Usually the connection with physical knowledge of material deterioration phenomenon and the behavior of $\phi_{ik}(x)$ suggests the use of a distribution belonging to the Weibull family (Garavaglia *et al.* 2004).

4 MAINTENANCE PLANNING

The results of the probabilistic analysis can also allow to plan a rehabilitation of the structure in order to achieve a given design value of the structural lifetime. The reliability function $R(t)=[1-P_f(t)]$ of the rehabilitated structure can be obtained by superposing the initial reliability $R_0(t)$ and its modifications $\Delta R_q(t)$ associated with the subsequent interventions $q=1,\ldots n$ (Kong and Frangopol 2003):

$$R(t) = R_0(t) + \sum_{q=1}^{n} \Delta R_q(t) \quad (8)$$

Different maintenance scenarios with essential and/or preventive interventions leading to different $\Delta R_q(t)$ can be considered in order to achieve the desired value of service life. For each scenario the total cost of maintenance C can be evaluated by summing the costs C_q of the individual interventions:

$$C=\sum_{q=1}^{n}\frac{C_q}{(1+v)^{t_q}}=\sum_{q=1}^{n}C_{0q} \tag{9}$$

where the cost C_q of the q^{th} rehabilitation has been referred to the initial time of construction by taking a proper discount rate v into account (Kong and Frangopol 2003).

In this way, based on a comparison among the costs of maintenance associated with different scenarios, a proper rehabilitation strategy can be finally selected.

5 APPLICATION

5.1 *Geometrical and Material Data*

The application refers to a bridge pier with the cross-section shown in Figure 2.a (Martinez y Cabrera 2002). The cross-section has main nominal dimensions d_y=8.20 m and d_z=9.00 m. It is reinforced with 160+248=498 bars having nominal diameters ∅=18 mm and ∅=30 mm, respectively, as shown in Figure 2.b. For concrete, the stress-strain diagram is described by the Saenz's law in compression and by an elastic perfectly plastic model in tension, with the following nominal parameters: compression strength f_c=–30 MPa; tension strength $f_{ct}=0.25|f_c|^{2/3}$; initial modulus $E_{c0}=0.25|f_c|^{1/3}$; peak strain in compression ε_{c0}=–0.20%; strain limit in compression ε_{cu}=–0.35%; strain limit in tension $\varepsilon_{ctu}=2f_{ct}/E_{c0}$. For steel, the stress-strain diagram is described by an elastic perfectly plastic model in both tension and compression, with the following nominal parameters: yielding strength f_{sy}=500 MPa; elastic modulus E_s=206 GPa; strain limit ε_{su}=1.00%. Additional details can be found in Biondini et al. (2004*b*, 2004*d*).

5.2 *Evolution of the Deterioration Process*

The structure is considered to be immersed in an aggressive environment. Figure 2.c shows the location of the aggressive agent, with concentration $C(t)=C_0$ along the external perimeter of the cross-section and $C(t)=½C_0$ along the internal one. The diffusion process of the agent within the structure is simulated by using cellular automata. The cellular automaton consists of a regular uniform grid of cells, with a discrete variable $C_i^k = C(\mathbf{x}_i, t_k)$ in each cell which represents the concentration of the component in the cell i at time t_k. The parameters of the automaton are chosen to regulate the process according to a given diffusivity $D = 10^{-11}$ m^2/sec and in such a way that the Fick's laws are satisfied (Biondini *et al.* 2004*c*).

Structural damage is modeled by introducing a degradation law of the effective resistant area for both the concrete matrix and the steel bars. Such damage is correlated to the diffusion process by assuming, for both materials, a linear relationship between the rate of damage $q = q(\mathbf{x}, t)$ and the mass concentration $C = C(\mathbf{x}, t)$ of the agent. In particular $q_c=(C_c\Delta t_c)^{-1}$ for concrete and $q_s=(C_s\Delta t_s)^{-1}$ for steel, where C_c and C_s represent the values of constant concentration which lead to a complete damage of the materials after the time periods Δt_c and Δt_s, respectively (Biondini *et al.* 2004*c*). In the following, the nominal values $C_c=C_s=C_0$, Δt_c=5 years and Δt_s=7.5 years are assumed.

Several parameters could be adopted as suitable measures for the probabilistic structural performance (Biondini *et al.* 2004*d*). In this study, the resistant bending moment M_y along the bridge axis under the axial force N =–100 MN is considered.

5.3 *Probabilistic Investigation and Simulation of the Monitoring Action*

The probabilistic model assumes as random variables the material strengths f_c and f_{sy}, the coordinates (y_p, z_p) of the nodal points p=1,2,… which define the two-dimensional model of the concrete cross-section, the coordinates (y_m, z_m) and the diameter $\varnothing_m$ of the steel bars m=1,2,…, the diffusivity coefficient D and the damage rates $q_c=(C_c\Delta t_c)^{-1}$ and $q_s=(C_s\Delta t_s)^{-1}$. These variables are assumed to have the probabilistic distribution with the mean μ and standard deviation σ values listed in Table 1.

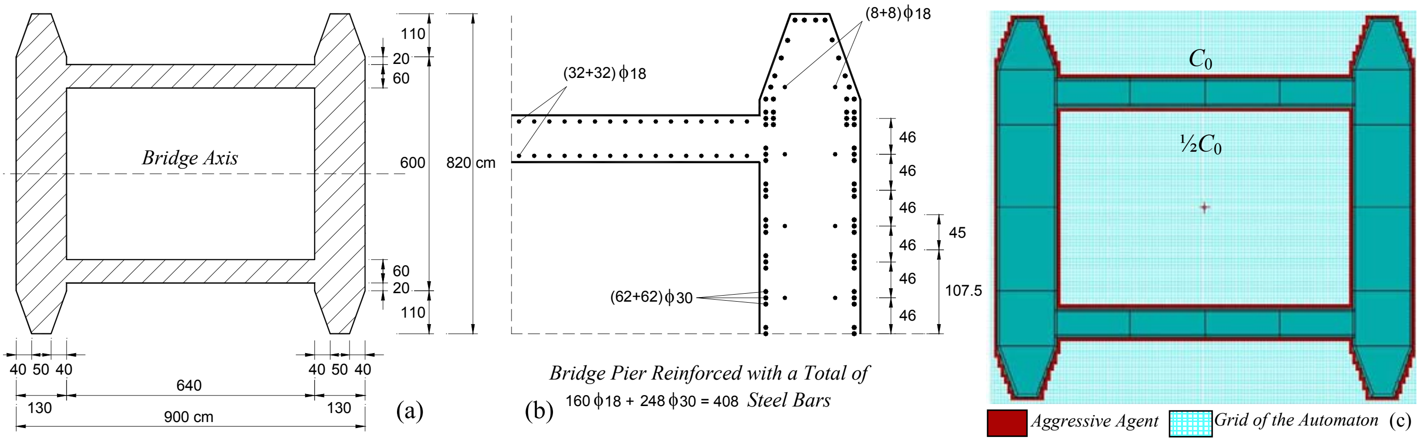


Figure 2. Box cross-section of a bridge pier. (a) Geometry and (b) location of the reinforcement. (c) Discretization of the structural model, grid of the cellular automaton and location of the aggressive agent.

Random Variable (t=t_0)	Distribution Type	μ	σ
Concrete strength, f_c	Lognormal	$f_{c,nom}$	5 MPa
Steel strength, f_{sy}	Lognormal	$f_{sy,nom}$	30 MPa
Coordinates of the nodal points, (y_i, z_i)	Normal	$(y_i, z_i)_{nom}$	5 mm
Coordinates of the steel bars, (y_m, z_m)	Normal	$(y_m, z_m)_{nom}$	5 mm
Diameter of the steel bars, $\varnothing_m$	Normal	$\varnothing_{m,nom}$	$0.10\varnothing_{m,nom}$
Diffusivity coefficient, D	Normal	D_{nom}	$0.10\ D_{nom}$
Concrete damage rate, q_c	Normal	$q_{c,nom}$	$0.30\ q_{c,nom}$
Steel damage rate, q_s	Normal	$q_{s,nom}$	$0.30\ q_{s,nom}$

Table 1. Probability distributions and their parameters (Biondini et al. 2004*b*, 2004*d*).

State Thresholds i	0	1	2	3	4	5
Threshold Value $m_{y,i}$	0.106	0.098	0.093	0.089	0.085	0.082
Threshold Amplitude $\Delta m_{y,i}$	–	0.008	0.005	0.004	0.004	0.003

Table 2. Thresholds values and amplitude of the states of the structural performance.

A Monte Carlo simulation is then carried out with a sample size suitable to obtain a stable statistical representation of the results. With reference to a sample of about 1000 simulations, Figure 3 shows the time evolution of the statistical parameters (mean value μ, standard deviation σ, minimum and maximum values of the 1000 outcomes) of the dimensionless values of the resistant bending moments $m_y=M_y/(|f_c|A_{c0}d_z)$ (Biondini *et al.* 2004*b*, 2004*d*).

In the following, the results of this simulation will be assumed as a large sample of experimental data that fully characterize the actual evolution of the structural performance. In this way, a set of simulated monitoring data will be extracted from such database in order to apply the proposed semi-Markov model. On the other hand, a direct comparison of the prediction so obtained with the full data set will allow to check the effectiveness of the proposed procedure.

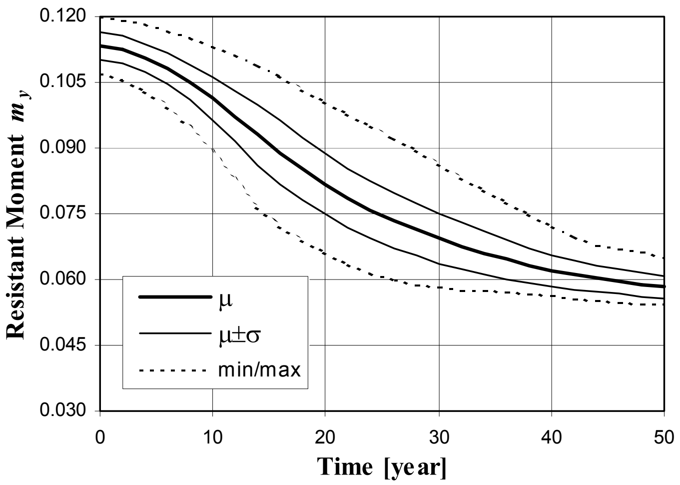


Figure 3. Time evolution of the resistance bending moment m_y: mean μ (thick line), standard deviation σ from the mean μ (thin lines), minimum and maximum values (dotted lines).

5.4 *Assessment of the Structural Reliability*

The evolution of the probabilistic performance of the pier cross-section is now investigated with reference to a different modeling aimed to predict the service life on the basis of a few monitoring data.

The random variable chosen to describe the structural performance of the structural system is the resistant bending moment m_y. In order to model the time evolution of such variable within a semi-Markov modeling, the following assumptions are introduced (Biondini *et al.* 2004*a*):

– The structure is undamaged at the initial time t_0=0.
– The damaged structure is considered to be in a state $i>0$ when $m_{y,i} \le m_y \le m_{y,(i-1)}$, where $m_{y,(i-1)}$ and $m_{y,i}$ are the upper and lower thresholds, respectively, which characterize the state i (Fig. 4).
– The structure evolves from a state $i>0$ to another state $k>i$, characterized by a lower level of performance $m_{y,k}<m_{y,i}$, during a time interval τ_{ik} (Fig. 4). Of course, the condition $k<i$, with $m_{y,k}>m_{y,i}$, is also possible if some maintenance is operated.

Under the hypothesis of s-MP, the time evolution of the structural behavior is then represented as transitions between different states of performance. The threshold values $m_{y,i}$ are listed in Table 2.

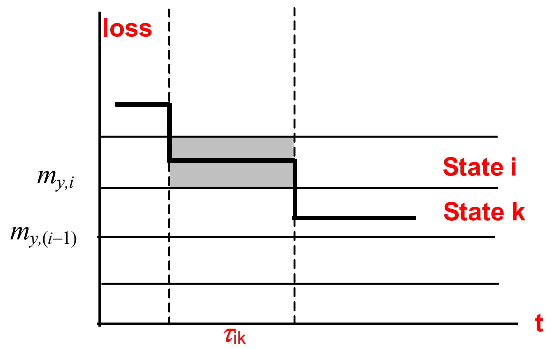


Figure 4. Transitions between different states of performance.

For each transition, the waiting time τ_{ik} must be modeled by choosing an appropriate p.d.f. As said before, this choice should be made on the basis of physical knowledge of phenomenon and on the characteristic of the distributions in the tail of the distribution where, usually, no much data are present. Based on such considerations, in the present application a Weibull distribution is chosen:

$$f_{ik}(t) = w_{ik}(t) = \alpha_{ik}\rho_{ik}(\rho_{ik}\ t)^{\alpha_{ik}-1}\exp[-(\rho_{ik}\ t)^{\alpha_{ik}}] \qquad (10)$$

where α_{ik} and ρ_{ik} are the shape parameters of each distribution, that in this application are estimated by the maximum likelihood criterion and Rosenbrock's optimization method.

It is worth noting that the effectiveness of the model depends on the monitoring time interval Δt. In fact, the transition from a state i to a state k could be not detected if $\tau_{ik} < \Delta t$, as shown in Figure 5. Therefore, for transitions with $k > (i+1)$ the following weighted superposition of the corresponding Weibull distributions should be adopted (Biondini *et al.* 2004*a*):

$$f_{\tau_i}(t) = \sum_k f_{ik}(t)p_{ik} = \sum_k w_{ik}(t)p_{ik} \qquad (11)$$

Based on the above criteria, in the following the proposed predictive model is applied to a sample of 20 data obtained by a Montecarlo simulation and simulating a real monitoring action.

Figure 6 shows the computed probability $\Pi_{1j}(t)$ of crossing the threshold j (with j=2,…6) if the system is in the state 1. The direct comparison of such distributions with the *statistical truth* represented by the results of the Monte Carlo simulation highlights the effectiveness of the proposed procedure.

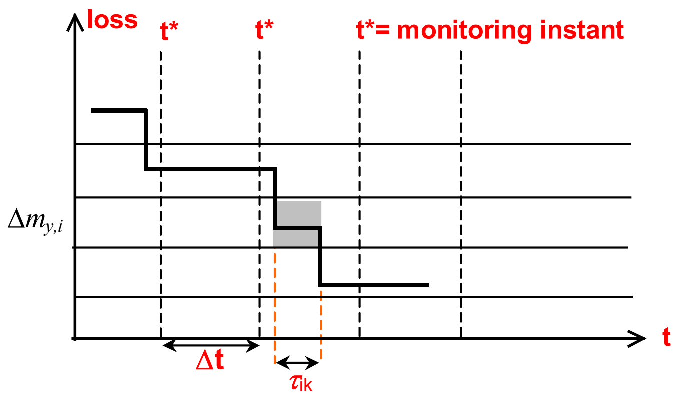


Figure 5. Transitions between different states of performance.

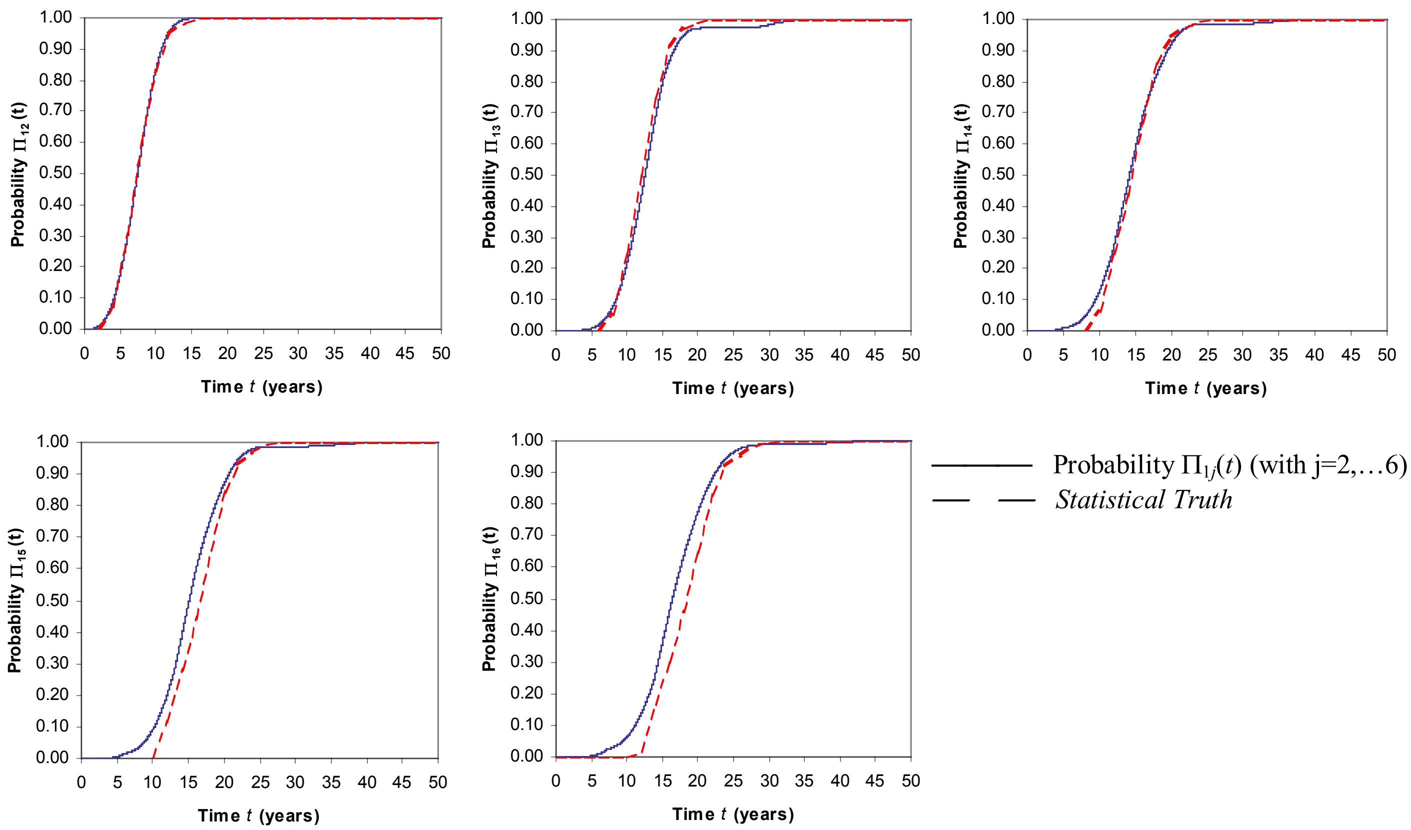


Figure 6. Comparison between the computed probability of failure $\Pi_{1j}(t)$ (with j=2,…6) and the *Statistical Truth*.

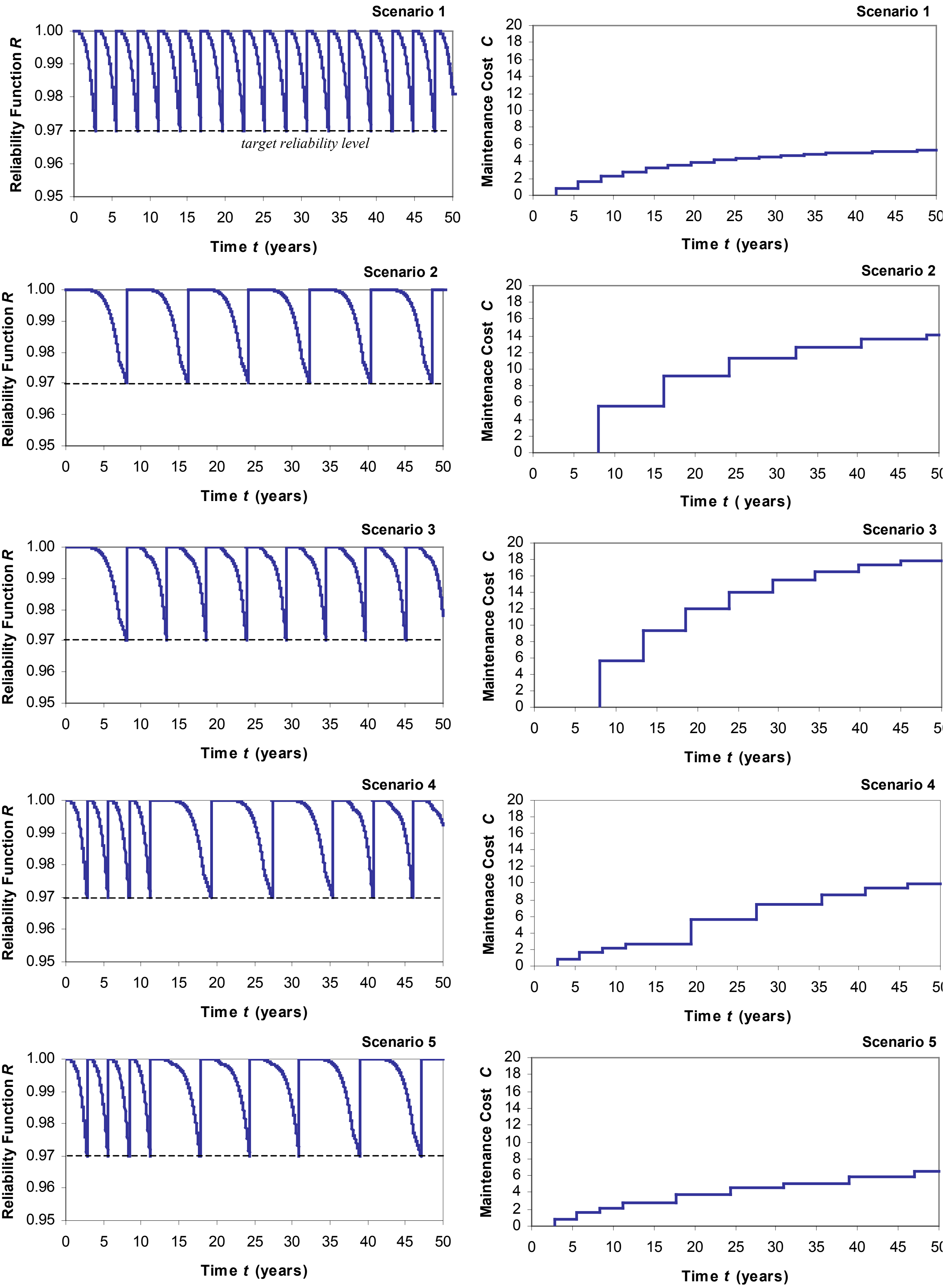
5.5 *Selection of the Maintenance Scenario*

The results of the previous analysis are now used to investigate the effects of maintenance interventions. With reference to a target reliability level R^*=0.97, Figure 7.a shows the reliability function $R(t)$ for five different scenarios of *preventive* maintenance. In all scenarios, when the target threshold R^* is reached a rehabilitation is made in order to increase the structural performance up to a prescribed threshold.

In the first scenario interventions are operated when the system reaches threshold 1 (state 2) to restore the initial performance associated to threshold 0 (state 1).

The second scenario assumes interventions applied when the system reaches the threshold 5 (state 6) and the initial performance associated to the threshold 0 (state 1) is restored.

The third scenario is similar to the second one, but each intervention is aimed to restore the level of performance associated to threshold 1 (state 2).



(a) (b)

Figure 7. (a) Time evolution of reliability function P_f and possible maintenance scenarios leading to a structural lifetime $T \geq 50$ years. (b) Time evolution of the normalized cumulative cost of the different maintenance scenarios of structural rehabilitation.

The fourth scenario follows a mixed strategy. For about the first 11 years maintenance interventions are applied when the system reaches the threshold 1 (state 2) and the initial performance associated to the threshold 0 (state 1) is restored. Subsequently, the system is allowed to reach the threshold 5 (state 6) and interventions are operated to restore the level of performance associated to the threshold 0 (state 1) for the subsequent 25 years, and the threshold 1 (state 2) until 50 years of lifetime.

The last scenario also presents a mixed strategy. For the first 11 years maintenance is operated as for the fourth scenario. After that, interventions are operated when the system reaches the threshold 2 (state 3) to restore the initial performance associated to the threshold 0 (state 1). Finally, the system is allowed to reach the threshold 5 (state 6) and interventions are aimed to restore again the level of performance associated to the threshold 0 (state 1) until 50 years of lifetime.

Figure 7.b shows the time evolution of the cumulative cost C corresponding to each maintenance scenario. The cumulative cost is normalized to the cost C_P of the single basic intervention associated to the transition from the state 2 to state 1. By denoting with C_{ij} the cost of the maintenance intervention associated to the transition from the state j to state i, the following cost ratio are assumed: $C_{12}/C_P=1$; $C_{13}/C_P=3$; $C_{16}/C_P=9$; $C_{26}/C_P=8$. A discount rate of money $\nu=0.06$ is assumed.

A direct comparison among the investigated scenarios shows that the more economical strategy involves a very low amount of allowed damage and a series of frequent small maintenance interventions aimed to restore the initial performance (see scenario 1). Large amounts of damage with early heavy interventions are usually very costly (see scenarios 2 and 3). However, they can become also convenient in a mixed strategy where the amount of allowed damage grows during time, involving in such way small interventions during the first years of lifetime and progressively heavier interventions as the age of the structure increases (see scenarios 4 and 5).

6 CONCLUSION

Data required for service life assessment of structural systems in aggressive environments can be often found from monitoring activities. However, for several reasons, such activities tend to be very limited in space and time and only a few observed data are usually available. In this paper, a probabilistic model aimed to assess the service life of deteriorating structures by using only a few measurements has been presented. Damage evolution has been modeled as a semi-Markov process with one step of memory and its formulation has been also exploited to account for eventual improvements of the structural performance. In this way, the proposed model has been also used as a tool to plan maintenance and/or rehabilitation interventions. The application to the prediction of the service life of a bridge pier and to the selection of a suitable preventive maintenance scenario highlighted the effectiveness of the procedure. In particular, the obtained results show that the proposed methodology is able to give reliable prediction of the expected service life even with a few measurements (20 observed data), and that it is able to give very useful information for the selection of an optimal maintenance strategy.

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