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On the Structural Analysis and Design of Cable-Stayed and Suspension Bridges

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ABSTRACT: This paper investigates the influence of the main parameters of the structural system on the structural stability and post-critical behavior of cable-supported bridges. The nonlinear equilibrium paths are carried out through a numerical model suitable to deal with large displacements and large strains. A series of comparative studies examines the influence of the cable layout, of the stiffness of the main structural elements and of particular anchor configurations with reference to cable-stayed and suspension bridges.

1 INTRODUCTION

The design of a new structure starts from basic criteria which, in essential form, resume its overall behavior and allows us to define a first set of dimensions and of thicknesses.

Such criteria derive from the capacity of perceiving and of highlighting the flow of the force vectors, which convey the applied loads to a fixed reference system ("to the earth"), as well as from specialized theories, which focus on the main factors characterizing the response of the structure to given loading conditions. Usually these criteria are presented as the "conceptual design" of that type of structure.

These specialized theories introduce simplified hypotheses, suitable to reduce the full stress behavior to a rational and more essential scheme and to translate the structural problem into analytical linear and non linear algorithms, which can be frequently solved in closed form. In conclusion these theories frame into essential schemes the suggestions of the intuition and of the experience and classify, in this way, well defined and known behaviors.

From this point of view, the design based on consolidated schemes can only continue to reproduce the same forms with great or little variants. For new and unconventional schemes we can not rely on similar references. Nowadays this is not a real problem, because computational structural analysis can help us to solve any kind of structure. Numerical solutions, however, give us particular results, linked to that particular structure. Parametrical studies can be developed for a wide variety of configurations, but they are not able to synthesize the essence of the structural behavior with the same efficiency of the classical theories. Furthermore, the usual parametric studies are carried out in the field of linear problems: they can only describe a limited range of the overall behavior of a structure and can suggest satisfactory results and measure equivalent performances even for very different typological configurations but, at the same time, these studies ignore fundamental differences which can develop along the load path to the collapse. Moreover the elastic analysis cannot allow us to measure how near we are to the limit or to collapse conditions, or to know how the collapse mechanism will be reached.

Systems which show marked non linear behaviors which are far from being linear are those suspended by cables. In particular, the structural response behavior of long span cable-supported bridges is strongly influenced by the characteristics of the cable layout and of the anchorage system.

By taking into account the large amount of geometrical, topological and stiffness characteristics which influence the actual behavior of the bridge, a systematic structural analysis can be carried out only through non linear numerical methods.

A numerical model suitable to deal with large displacements and large strains, typical of these system, has been developed. Through the principle of virtual works, developed according to the Updated Lagrangian formulation, a truss and a beam element have been introduced.

The extension of such a formulation to complex structures gives a central role to the numerical techniques for solving non linear problems in presence of singular points and of unstable behaviors. As known, the classical Newton-Raphson Method allows us to follow the equilibrium paths working by load increments, but it is not able to follow post critical behaviors. In order to extend the analysis to the post critical field, the arc length method has been used. The method has been improved by specific procedures, able to detect the bifurcation points and the paths which depart from these points. This numerical approach has been tested through many benchmarks.

In this paper a series of comparative studies examines the influence of the cable layout, of the stiffness of the main structural elements and of particular anchoring configurations with reference to cable stayed and suspension bridges.

2 CABLE-STAYED BRIDGES

We refer to the cable stayed bridge geometry shown in Figure 1, having the mechanical and geometrical characteristics listed in Table 1. In the same table the stay pretensioning is given. In the following, the influence of the cable layout on the overall bridge stability is investigated. To this aim, we intend to study the cases described in Table 2.

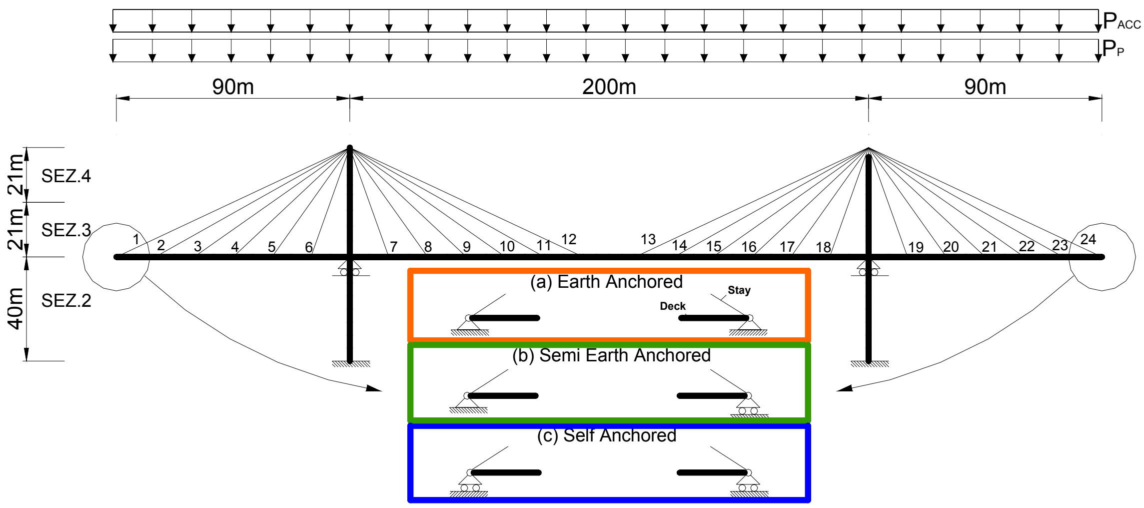

Figure 1. Structural model of the cable-stayed bridge.

Table 1. Mechanical and cross-sectional properties, and pretensioning of the stays.

Element	E [GPa]	I_x [m^4]	I_y [m^4]	A [m^2]	w [kN/m]
Deck	40	4.0896	236.09	8.0759	100
Pylon(0-40m)	40	1.3482	13.843	5.00	–
Pylon(40-61m)	40	0.722	2.7750	2.70	–
Pylon(61-82m)	40	0.6219	2.6417	2.30	–
Stay(1 e 2)	210	–	–	0.01	–
Stay(3,4,5,6)	210	–	–	0.005	–

Pretensioning of the stays						
Stay	1≡24	2≡23	3≡22	4≡21	5≡20	6≡19
$P_{N0,i}$[kN]	4340	3480	2522	2219	1840	1590

Table 2. Case studied for cable stayed bridges.

Model	Description
(a)	Reference case: Fan shaped cable stayed bridge, with earth anchored lateral stays.
(b)	Model (a) with roller support at right end (semi-earth anchored system).
(c)	Model (a) with both ends roller supported. The lateral stays are anchored at the deck (self-anchored system).
0	Reference case: Fan shaped cable stayed bridge, semi-earth anchored.
1-6	The stays, which in Model 0 converge to the top of the antennas, are no longer anchored all in one point but reach the antenna with variable distances within each other.
7	Model 0, with stays having such a distance within each other as to reach the harp configuration.
(d)	Fan shaped cable stayed bridge, with self-anchored lateral stays and deck flexural stiffness equal to a quarter of that of the starting Model 0.
(e)	Harp shaped cable stayed bridge, with self-anchored lateral stays and deck flexural stiffness equal to a quarter of that of the starting Model 0.

2.1 *Comments on the behavior of (a), (b), (c) Models (Figure 2)*

The earth anchored Model (a) is stable and behaves like a typical tension hardening system. The external cables (Cable 1 and Cable 24) constrain the heads of the antennas against longitudinal displacements and convey the horizontal components of the stay tension directly to the earth.

Model (b) presents a limit point (λ = 15.73) caused by the instability of the right antenna, which is now free to move, while Cable 24 loads the deck, contributing to cause large longitudinal and vertical displacements.

Model (c) has a limit point (λ = 15.48) which, in this particular case, is a bit higher than for Model (b), but it presents a worse post critical behavior without recovering stiffness after having reached the limit point.

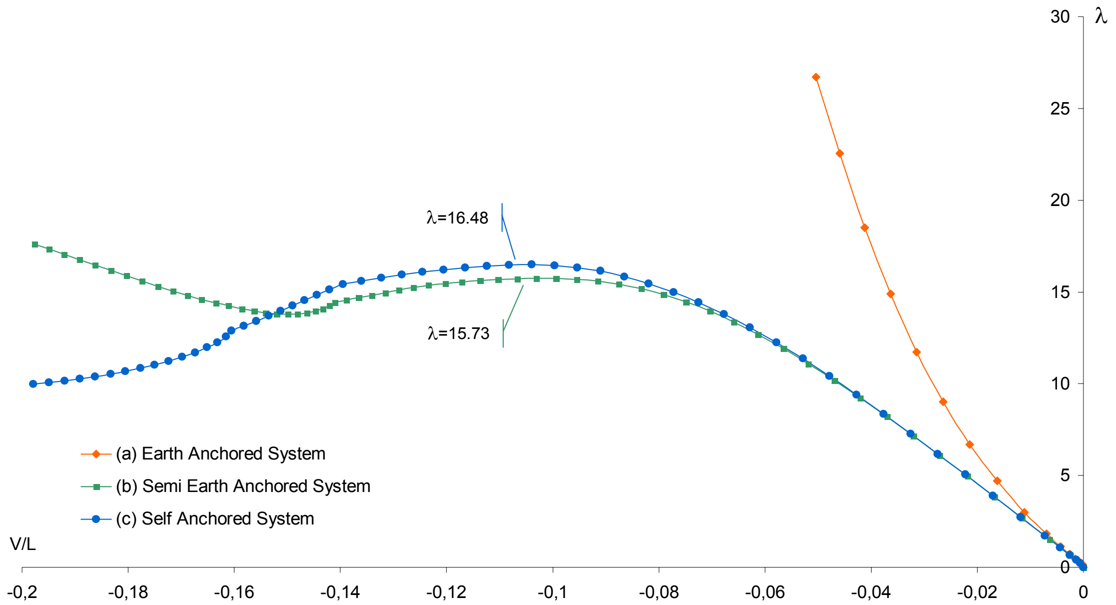

Figure 2. Live-load deflection path of cable-stayed bridges with different typology of supports.

2.2 *Comments on the behavior of 0 ÷ 7 Models (Figure 3)*

The sequence of Models 0-7 realizes the gradual development of stay geometry from pure Fan to a pure Harp configuration. For each of these Models, Figure 3 shows the load deflection path associated to the vertical displacement at the middle of the central span.

For Models 0-3 we have increasing limits points (λ = 15.73, 16.67, 18.24), with a certain tension hardening post-critical branch and, for Model 1, a hint of snapback.

For Models 5-7 there are no limit points and the system appears stiffer than in the previous cases.

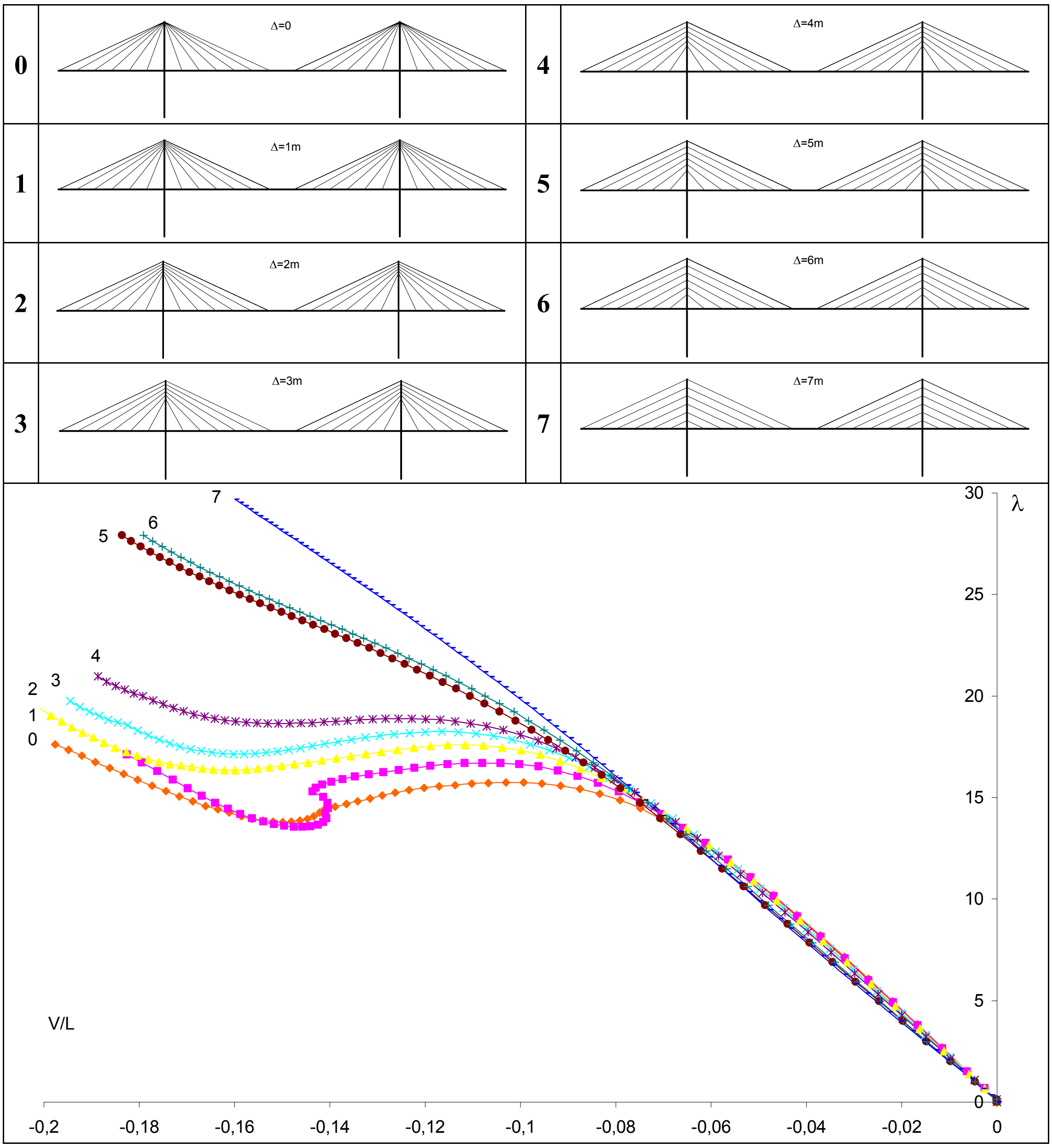

Figure 3. Evolution of the live load-displacement path with the variation of the configuration of the suspension system from the Fan to the Harp arrangement.

2.3 *Comments on the behavior of (d)-(e) Models (Figures 4 and 5)*

It must be remembered that the sequence of results shown in Figure 3 is specialized to a particular combination of dimensions and of stiffness characteristics. From these results, for instance, it appears that there is always a limit point for those systems having the cables converging at the top of the antennas, while for Harp type systems the behavior appears to be stable and tension hardening. This can be simply explained by observing that in the Fan case the stay action results in a concentrated force acting at the top of the antenna, while in the Harp system the vertical components of the stay action are distributed along a certain height (Figure 4.a).

In reality, for usual dimensions and stiffness distributions, is the Harp system the critical one. In fact, in a Fan system the applied load is transferred through the CB stay to the three BOD bar system and, in this way, to the rigid supports (Figure 4.b). In the Harp system the applied load acts directly on the CBAD system and it is transferred to the supports involving also the flexural stiffness of the deck. Moreover in the Harp systems the stays have a lower vertical slope than for the Fan and this provokes a higher compression force in the deck. Figure 5 compares the load deflection path for the two cases (d-Fan) and (e-Harp).

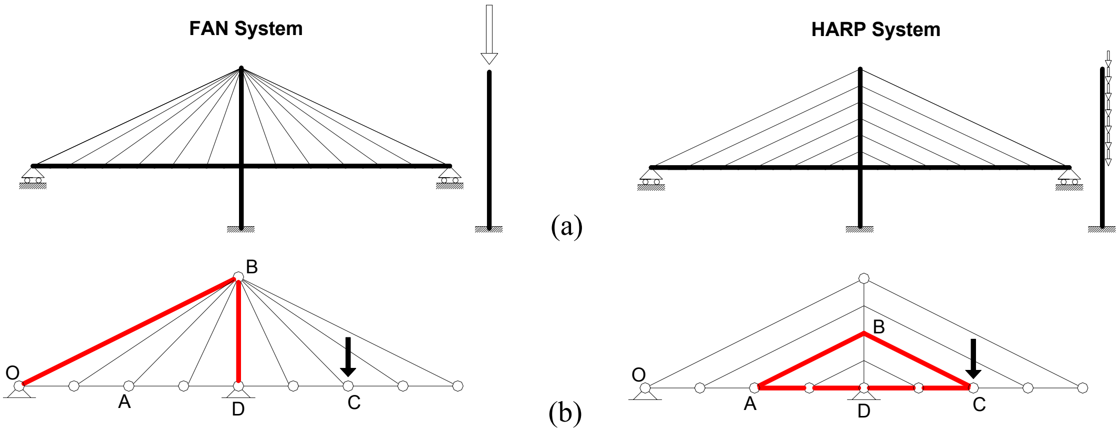

Figure 4. Role of the suspension system in the structural stability of (a) the pylons and (b) the deck.

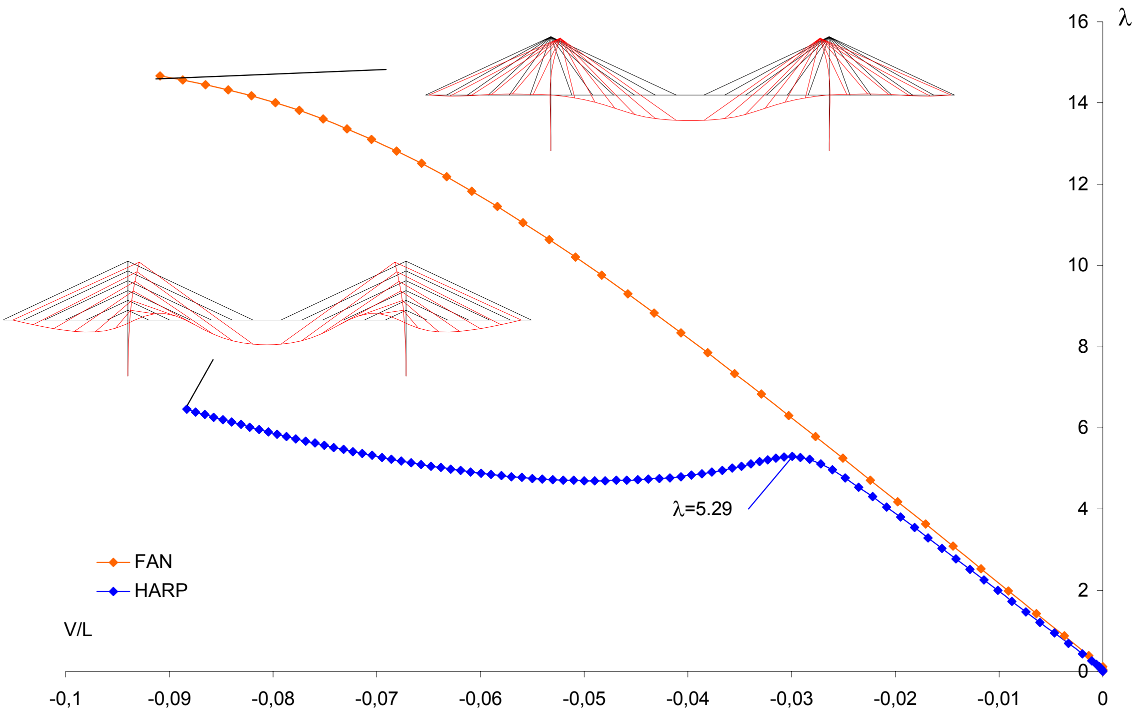

Figure 5. Load deflection path of the Fan and Harp systems with low stiffness of the deck.

3 SUSPENSION BRIDGES

In suspension bridges, the forces acting on the main cable, can be transferred to the earth directly through anchor blocks at the end of the side spans or can be self-contained by the longitudinal action of the deck (self-anchored systems). The self-anchored systems do not require the volumes of the anchor blocks. On the other hand the stiffening girder, heavily compressed, will require a larger cross section.

3.1 *The case of the Great Belt Bridge*

The present application refers to the basic dimensions of the Great Belt Bridge, shown in Figure 6. The bridge has a central span length $s_c = 1624$ m, two lateral spans lengths $s_l = 535$m, height of the pylons h = (180+77.6) = 157.6 m, sag of the cable f =180 m. Based on such scheme, a comparative study has been carried out by varying some fundamental characteristics of the bridge. In particular, the cases listed in Table 3 have been examined.

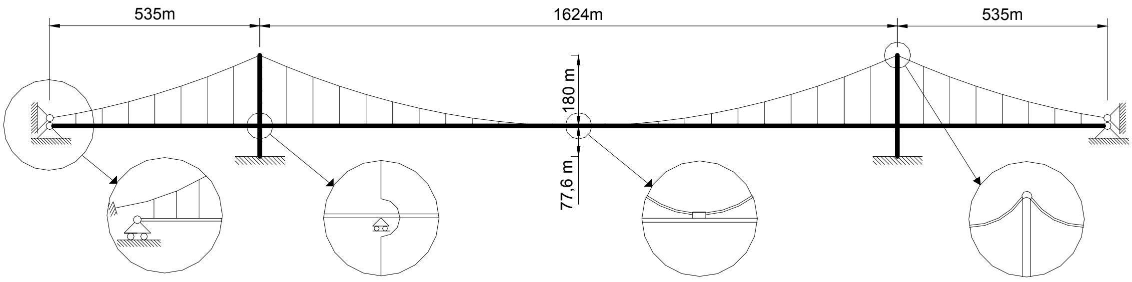

Figure 6. Great Belt Bridge. Structural scheme and basic dimensions.

Table 3. Cases studied for suspension bridges.

Model	Description
0	Real case: suspension bridge earth anchored (stable).
1	The bridge of model 0, but self-anchored. (Area and Inertia of the deck are $A=0.5m^2$, $J=1.66m^4$).
2	The bridge of Model 1 with increased deck stiffness ($A = 0.9\ m^2$, $J = 3.32\ m^4$).
3	The bridge of Model 1 with increased deck stiffness ($A = 1.4\ m^2$, $J = 4.98\ m^4$).
4	The bridge of Model 1 with height pylons increased from 257.6m to 277.6m, but maintaining the sag of the cable f = 180 m.
5	The bridge of Model 4 with increased deck stiffness ($A = 0.8\ m^2$, $J = 4.0\ m^4$).
6	Equal to Model 1, but having a truss hanger net.
7	Equal to Model 1, but with hangers having a variable slope along the side spans.
8	Equal to Model 7 with increased deck stiffness ($A = 0.8\ m^2$, $J = 3.32\ m^4$).

The results are shown in Figure 7. The first diagram presents the values of the load multiplier at the limit point in function of the inertia of the deck. The little circles used to mark the limit points allow us to recognize if a limit point is stable or if it is followed by unstable branches.

The main conclusions of these comparative studies can be synthesized as follows:

- Suspended, earth anchored systems are generally stable.
- The behavior of suspended, self-anchored systems is sensitive to the action of the deck and to the slackness of the hangers.

Furthermore, for suspended, self-anchored systems we can observe that:

- There are limit and bifurcation points which lead to post critical and generally unstable response, with snap back effects.
- The limit load multiplier can be augmented by stiffening the deck. However, it must be kept in mind that the increase in weight causes an increase in the axial force, with the possibility of a critical behavior taking place in the pylons and not in the deck.
- An increment of the pylon height involves a greater slope of the main cables: this involve that a greater portion of the total force in the cables is conveyed directly to the earth through the bearings, while a lower fraction of this force contributes to the axial component in the deck. This brings to a higher value of the load multiplier.
- The simultaneous increase of the deck inertia and of the pylons height do not produce relevant positive effects, because the critical behavior depends on the pylons again, while the piers are higher and more stressed.
- It is possible to obtain relevant increments in the limit load multiplier just by increasing the slope of the hangers in the lateral spans, without having any simultaneous increase in costs or in dead loads, as can be seen in models 7 and 8.

3.2 *The case of the Messina Strait Bridge*

A final example of application regards the influence of an elastic connection between the main cable and the anchor blocks of the Messina bridge. Figure 8 shows the model of the bridge. Figure 9 shows a family of curves, corresponding to different stiffness of the elastic connection, which show how the vertical displacements of the point at the top of the anchorage lever divided by the lever length (V/H), varies in function of a scalar multiplier λ of the uniform live load applied to the deck.

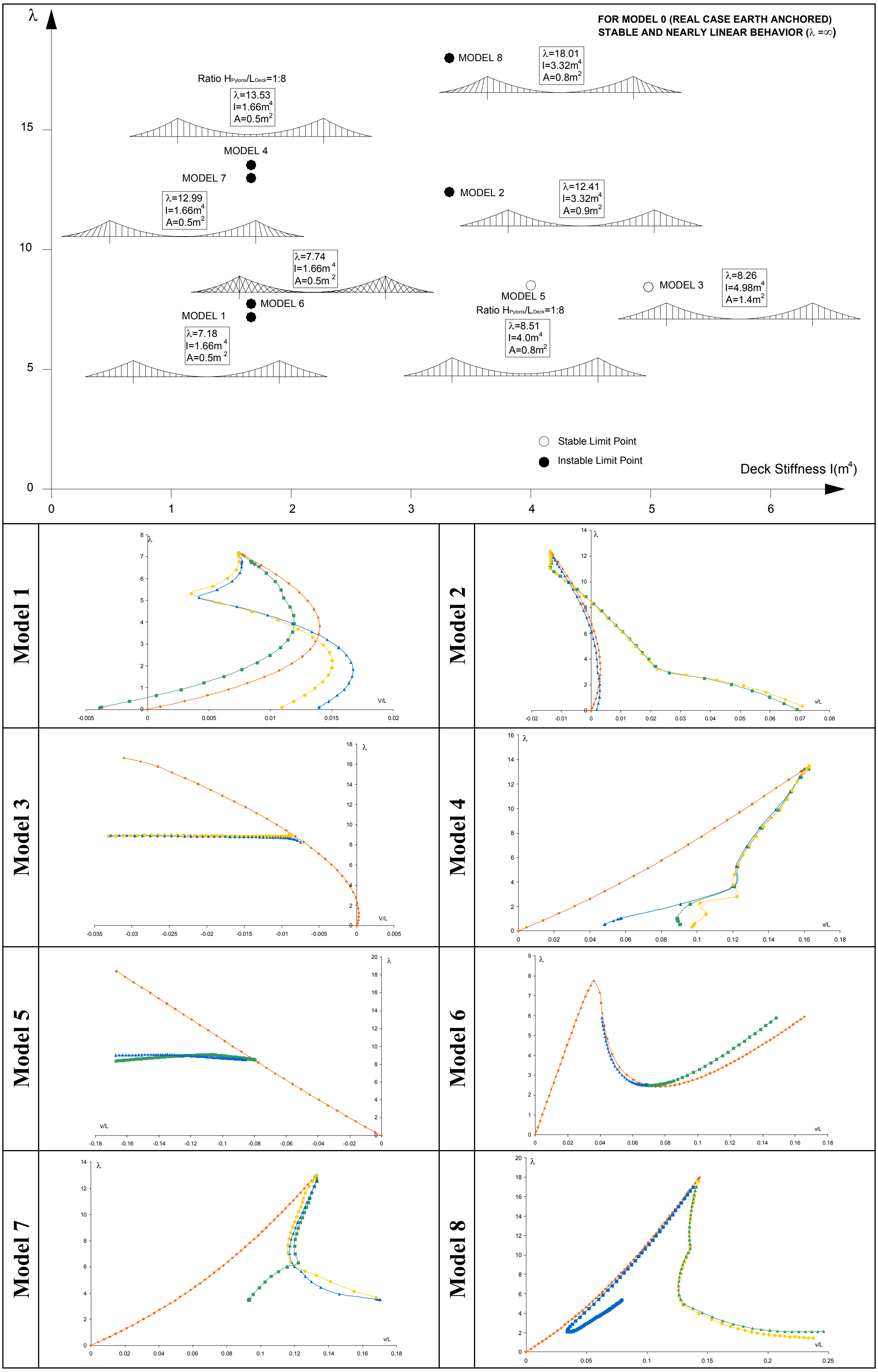

Figure 7. Great Belt Bridge. Live load-displacement paths for the models 1-8 described in Table 3.

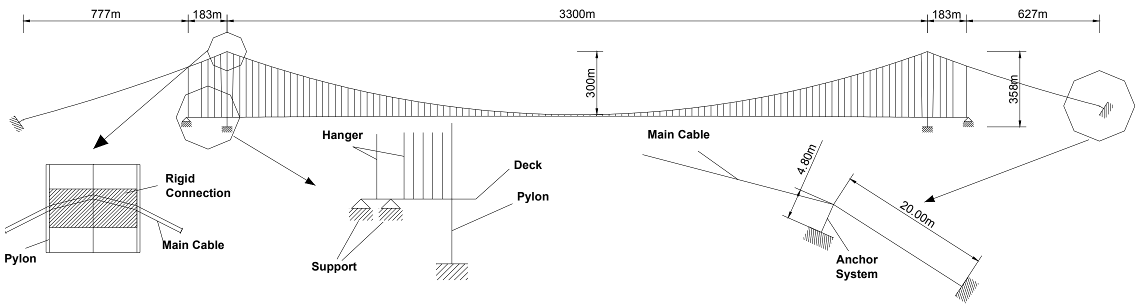

Figure 8. Structural Model of the Messina Strait Bridge.

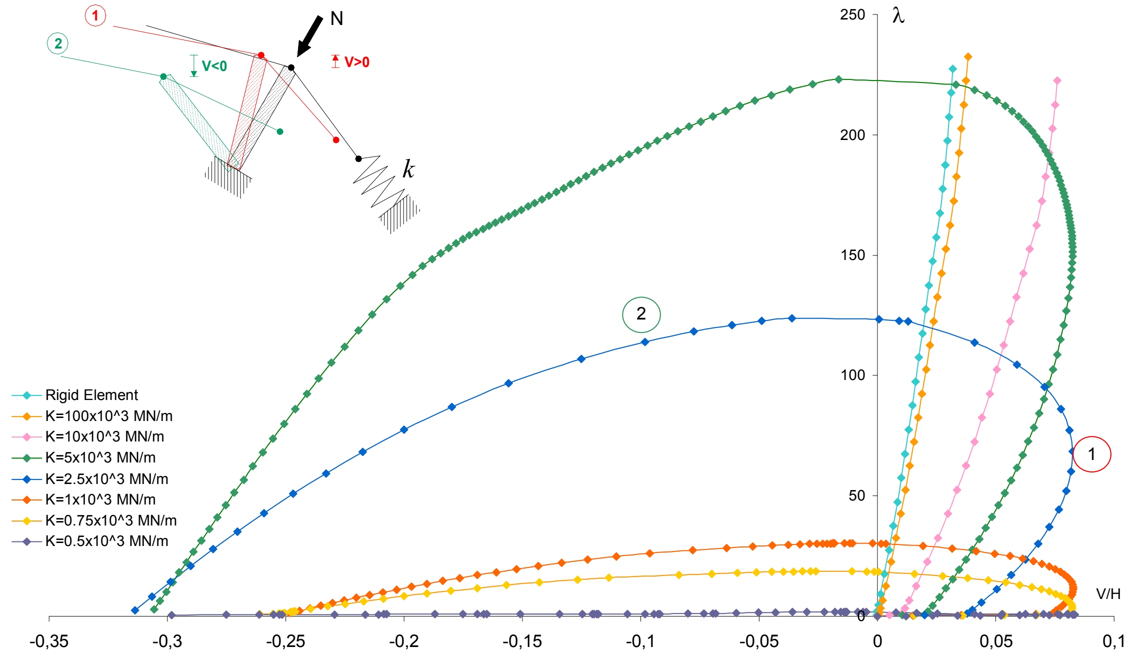

Figure 9. Messina Strait Bridge. Live load-displacement paths with respect to the ratio between the vertical displacement V of the point at the top of the anchorage lever and its height H (V/H).

4 CONCLUSIONS

The role played by the main parameters of the structural system in driving the structural stability and post-critical behavior of cable-supported bridges has been investigated. In particular, a parametric study based on a numerical model suitable to deal with large displacements and large strains has been carried out. The results of this study allow to highlight and quantify the high sensitivity of the structural response of cable-stayed and suspension bridge with respect to the characteristics of both the suspension and anchorage systems.

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