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FUZZY OPTIMUM DESIGN OF CONCRETE BRIDGES

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Abstract. *The fuzzy optimum design of concrete structures under static loads as regards both serviceability and ultimate limit conditions is presented. In this paper special attention is devoted to the optimal design of reinforced concrete bridges, but the proposed procedure is also suitable for other kind of structures. Design variables include either the shape and the dimensions of the concrete cross-sections, or the amount and location of the reinforcement. These quantities are assumed as deterministic. However, additional fuzzy variables are considered to take the uncertainties involved in the basic properties of the materials into account. The objective of the design process is to minimize the structural cost of the system according to side and behavioral constraints. An optimal solution is achieved for several levels of uncertainties by using genetic algorithms. The structural analyses needed for the solution process are performed by means of a composite finite beam element able to take mechanical and geometrical non-linearity into account. An application to the optimization of a reinforced concrete continuous bridge shows the effectiveness of the procedure.*

1 INTRODUCTION

In this paper, a fuzzy decision-making criterion is applied to the optimization of reinforced concrete (R.C.) framed structures. Special attention is devoted to the optimal design of R.C. bridges, but the proposed procedure is also suitable for other kind of structures. At first, a systematic approach to the optimal design of R.C. members under given load conditions is described (Biondini 1999). Design variables include either the shape and dimensions of the concrete cross-sections, or the amount and location of the reinforcement over the structure. Design constraints are directly applied on such variables accounting for both serviceability and ultimate limit conditions on the structural behavior. The objective of the design process is to minimize the structural cost of the system according to side and behavioral constraints.

Since the concrete dimensions and the location of the reinforcement are usually required as discrete quantities (f.i. whole centimeters) and the reinforcing bars are manufactured in standard sizes, the structural problem should be formulated by assuming the design variables as discrete-valued. Having stated the problem in such a way, in this work a genetic algorithm approach is selected as an efficient and robust search method (Michalewicz 1992). As mentioned, the fitness of each solution needs to be evaluated in according to the associated structural performances. In particular, strength and member's ductility values are obtained through a nonlinear structural analysis taking the constitutive laws of the materials (concrete and steel) into account. The method of analysis is based on the finite element technique and is performed by using a suitable R.C. beam element (Bontempi et al. 1995, Malerba 1998).

Conventional optimization considers all the relevant data as deterministic. As known, in the real life the design parameters which define the structural problem cannot be considered as deterministic. Where the data are random and the corresponding density functions are known, the methods of stochastic optimization should be applied. However, in several cases the data are not affected by randomness but are known only approximately. In such cases the imprecision of the data may be usefully treated through a fuzzy optimization approach (Biondini et al. 2000). In particular, after some quantities are fuzzified, a weak form of the fitness of the structural system is formulated by assuming suitable level of aspirations. Therefore, a new parameter which represent the grade of the membership functions is introduced and the problem becomes one of the minimization of the fuzzy structural cost with respect to the new fuzzy constraints. Such a parameter, being an index of the acceptability of that decision, can be considered as a fuzzy measure of the structural reliability. An application to a R.C. continuous bridge shows the effectiveness of the presented procedure.

2 FUZZY OPTIMUM STRUCTURAL DESIGN

2.1 Formulation of the optimization problem

The problem of the optimum structural design consists of finding a set of design variables (i.e. geometrical dimensions, amount and location of the reinforcement, etc.) which accounts for assigned design constraints (on stresses, strains, forces, displacements, etc.) and optimizes one or more given target requirement (structural cost, structural weight, strength, ductility, etc.). Therefore, from a mathematical point of view, the purpose of a one-target design process is to find a vector **x** which optimizes the value of an objective function $f(\mathbf{x})$, according to either side constraints with bounds $\mathbf{x}^-$ and $\mathbf{x}^+$, or inequality $\mathbf{g}(\mathbf{x})\leq\mathbf{0}$ and/or equality $\mathbf{h}(\mathbf{x})=\mathbf{0}$ behavioral constraints. Since, without any loss of generality, minimization problems only may be considered, the optimization problem is cast in the following form:

$$\min_{\mathbf{x}\in D} f(\mathbf{x}) \qquad D=\left\{ \mathbf{x} \mid \mathbf{x}^- \leq\mathbf{x}\leq\mathbf{x}^+,\ \mathbf{g}(\mathbf{x})\leq\mathbf{0},\ \mathbf{h}(\mathbf{x})=\mathbf{0} \right\} \tag{1}$$

which, in the general case, represents nonlinear programming problem.

2.2 Objective function

As already mentioned, several quantities able to represent the structural performances may be chosen as target requirements for the optimal design. In this work, the adopted objective function is related to the cost $C(\mathbf{x})$ of the structure. In particular, the sum of the costs of the component materials, concrete and reinforcing steel, is considered:

$$C(\mathbf{x})=c_c V_c + c_s V_s \tag{2}$$

where $V_c(\mathbf{x})$, $V_s(\mathbf{x})$, are the total volumes of the concrete and of the steel, respectively, and c_c, c_s, are the corresponding unit costs. It is clear that additional cost components (i.e. cost of formwork), may be considered in this formulation. However, the function $C(\mathbf{x})$ represents a consistent criterion to compare different designs rather than the actual structural cost.

Since unit costs are usually not depending on the vector **x**, the optimization problem is implicitly defined by the unit cost ratio $c=c_s/c_c$. Therefore, the objective function is assumed as follows:

$$f(\mathbf{x})=V_c + cV_s \tag{3}$$

2.3 Design constraints and penalty function

The dimensions and the components of the vectors $\mathbf{g}(\mathbf{x})$ and $\mathbf{h}(\mathbf{x})$ which define the behavioral constraints are clearly depending on the particular design problem which has to be solved. However, at this stage, it is important to outline that direct search strategies of the optimal solution, like genetic algorithms, often cannot directly take behavioral constraints into account. One way to handle such constraints is to penalize unfeasible vectors $\mathbf{x}\notin D$ by adding to the objective function a penalty term $p(\mathbf{x})\geq 0$, whose value is zero if no constraint violation occurs and is positive otherwise. In other words, the original constrained optimization problem is transformed into the following unconstrained form:

$$\min_{\mathbf{x}\in E} \varphi(\mathbf{x}) \qquad E=\left\{ \mathbf{x} \mid \mathbf{x}^- \leq\mathbf{x}\leq\mathbf{x}^+ \right\} \tag{4}$$

being $\varphi(\mathbf{x})=f(\mathbf{x})+p(\mathbf{x})$ the new penalized objective function which has to be optimized within the side-bounded, obviously convex, feasible domain $E\supseteq D$.

The choice of a suitable penalty function $p(\mathbf{x})$ represents a critical point in establishing the effectiveness and robustness of the search technique. In particular, to lead the procedure toward an optimal solution $\mathbf{x}\in D$ it has to be able and to assure a hierarchical arrangement of the explored set of possible solutions $\mathbf{x}\in E$. This becomes very important if ones use those type of optimization techniques, like genetic algorithm, which process contemporarily a population of possible solutions rather than a single candidate (see Biondini 1999 for details).

2.4 Handling fuzzy uncertainties by genetic algorithms

Due to the uncertainties involved in the design process, the geometrical and mechanical properties which define the structural problem should not be considered as deterministic quantities. Anyway, at a first stage of design, the unknown variables **x** can be identified with some characteristic values and treated as deterministic. In this context, a slightly more refined approach is still possible if the uncertainties on some other known properties of the system are taken into account, f.i. by using a fuzzy criterion. Therefore, by denoting with **y** a vector of fuzzy variables, the original optimization problem can be rewritten as:

$$\min_{\mathbf{x}\in D} f(\mathbf{x},\mathbf{y}) \qquad D = \left\{ \mathbf{x} \;\middle|\; \mathbf{x}^- \le \mathbf{x} \le \mathbf{x}^+,\ \mathbf{g}(\mathbf{x},\mathbf{y}) \le \mathbf{0},\ \mathbf{h}(\mathbf{x},\mathbf{y}) = \mathbf{0} \right\} \tag{5}$$

This problem may be solved by parametric programming, but such techniques are often computationally expensive. Simulation methods, usually more effective, can be certainly used, but also in this case the computational demand tends to be very hard if they work in series with the optimization process. Alternatively, an efficient parallel processing, which contemporarily search for an optimal solution **x** and accounting for the uncertainties in the system properties **y**, can be thought if a suitable optimization technique is selected. To this aim, genetic algorithms work very well since they process contemporarily a population of design points rather than a single point (Michalewicz 1992). This higher exploration of the search space, besides to give more chances to avoid local optimum, assures a lower sensitivity of the search process to the external noise which the uncertainties introduce. In this way, several best solutions can be exploit, one for each level of uncertainties.

Properly, genetic algorithms works on a *population* $\mathrm{X} = \{\, \mathbf{x}_1\ \mathbf{x}_2\ \dots\ \mathbf{x}_m \,\}$ of *m individuals* $\mathbf{x}_k\in E$ subjected to an evolutionary process where individuals compete between them to survive in proportion to their *fitness* with the *environment E*. In this process, population undergoes continuous reproduction by means of some *genetic operators* (basically selection, crossover and mutation) which, because of competition, tend to preserve best individuals. The fitness of each individual **x** is measured by a scalar function $F(\mathbf{x})\ge 0$ which increase with the adaptability of **x** to its environment *E*. Clearly, such function must takes also the fuzzy variability of **y** into account. To this aim, a particular outcome of **y**, selected on the basis of given membership functions, is associated to each individual **x**. Therefore, as regards the optimization problem, the fitness function is related to the penalized objective function $\varphi(\mathbf{x},\mathbf{y}) = f(\mathbf{x},\mathbf{y}) + p(\mathbf{x},\mathbf{y})$ as follows:

$$F(\mathbf{x},\mathbf{y}) = \frac{\varphi(\mathbf{x}_B,\mathbf{y}_B)}{\varphi(\mathbf{x},\mathbf{y})} \tag{6}$$

In this way, the fitness of the best individual $\mathbf{x}_B$ in the current generation equals unity, or $F(\mathbf{x})\in[0;1]$. However, to assure an appropriate hierarchical arrangement of the individuals, such function should be properly scaled. Details about the adopted scaling rules, internal coded representation of the population, genetic operators and termination criteria, can be found in a previous paper (Biondini 1999).

3 DESIGN CRITERIA AND STRUCTURAL ANALYSIS OF R.C. STRUCTURES

In most cases the R.C. structure should be analyzed by taking material and geometrical non-linearity into account and their performance should generally be described with reference to a specified set of limit states as regards both serviceability and ultimate conditions (Bontempi et al. 1998, Biondini et al. 2000).

3.1 Limit states and load multipliers

Splitting cracks and considerable creep effects may occur if the compression stresses σ_c in concrete are too high. Besides, excessive stresses σ_s either in reinforcing steel can lead to unacceptable crack patterns. Excessive displacements **s** may also involve loss of serviceability and then they have to be limited within assigned bounds $\mathbf{s}^-$ and $\mathbf{s}^+$. Based on these considerations, the following limitations account for adequate durability at the serviceability stage (*Serviceability Limit States*):

$$-\sigma_c \le -\alpha_c f_c \qquad |\sigma_s| \le \alpha_s f_{sy} \qquad \mathbf{s}^- \le \mathbf{s} \le \mathbf{s}^+ \tag{7}$$

where α_c and α_s are suitable reduction factors of the limit strengths f_c and f_{sy}.

When the strain in concrete ε_c or in the reinforcing steel ε_s reaches a limit value ε_{cu} or ε_{su}, respectively, the collapse of the corresponding cross-section occurs. However, this doesn't necessarily means the collapse of the whole structure, which is due to the loss of equilibrium arising when the reactions **r** requested for the loads **f** can no longer be developed. So, the following ultimate conditions have to be verified (*Ultimate Limit States*):

$$-\varepsilon_c \le -\varepsilon_{cu} \qquad |\varepsilon_s| \le \varepsilon_{su} \qquad \mathbf{f} \le \mathbf{r} \tag{8}$$

Since these limit states refer to internal quantities of the system, a check of the structural performance through a non-linear analysis needs to be carried out at the load level. To this aim, it is useful to assume $\mathbf{f} = \mathbf{g} + \lambda\mathbf{q}$, where **g** is a vector of dead loads and **q** a vector of live loads whose intensity varies proportionally to a unique multiplier $\lambda \ge 0$. With this position, the safe domains at both serviceability and ultimate states and the previous limit conditions can be synthetically formulated as follows:

$$\lambda \le \lambda_S = \max_{\lambda\in S}\lambda \qquad S = \{\ \lambda \mid -\sigma_c \le -\alpha_c f_c,\ |\sigma_s| \le \alpha_s f_{sy},\ \mathbf{s}^- \le \mathbf{s} \le \mathbf{s}^+\ \} \tag{9}$$

$$\lambda \le \lambda_U = \max_{\lambda\in U}\lambda \qquad U = \{\ \lambda \mid -\varepsilon_c \le -\varepsilon_{cu},\ |\varepsilon_s| \le \varepsilon_{su},\ \mathbf{f} \le \mathbf{r}\ \} \tag{10}$$

being λ_S and λ_U the limit multipliers which define the failure loads.

3.2 Structural model and non-linear analysis

The structural analyses needed to check the limit states and to evaluate the corresponding limit load multipliers are performed by means of a R.C. finite beam element whose formulation, based on the Bernoulli-Navier hypothesis, deals with mechanical and geometrical non-linearity (Bontempi et al. 1995, Malerba 1998). In particular, both material and geometrical contributes to the element stiffness matrix and to the nodal forces vector, equivalent to the applied loads, are derived by applying the principle of the virtual displacements and then evaluated by numerical integration over the length of the beam.

The previous computations are performed by assuming the displacement functions of a linear elastic beam element having uniform cross-sectional stiffness and loaded only at its ends (Przemieniecki 1968). However, due to material non-linearity, the cross-sectional stiffness distribution along the beam is non uniform even for prismatic members with uniform reinforcement. Thus, it has to be computed for each section by integration over the area of the composite element, or by assembling the contributes of both concrete and steel. To this aim, the non-linear constitutive laws of the materials must be defined.

Finally, by assembling the stiffness matrix **K** and the vectors of the nodal forces **f** in a global coordinate system, the equilibrium of the whole structure is formally expressed as follows:

$$\mathbf{K}\mathbf{s}=\mathbf{f} \tag{11}$$

where **s** is the vector of the nodal displacements. It is worth noting that the vectors **f** and **s** have to be considered as total or incremental quantities depending on the nature of the stiffness matrix **K**=**K**(**s**), or if a secant or a tangent formulation is adopted.

4 APPLICATION AND CONCLUDING REMARKS

The presented procedure is applied to the optimal design of the continuous R.C. bridge shown in Figure 1, with: $l = 30.00$ m, $b_{top} = 13.00$ m, $b_{bot} = 7.00$ m, and $b_{web} = 0.40$ m.

The stress-strain diagram for concrete is described by the Saenz’s law in compression and by an elastic perfectly-plastic model in tension. It is completely defined by the following quantities: compression strength $f_c = -40$ MPa, tension strength $f_{ct} = 2.92$ MPa, initial modulus $E_{c0} = 35$ GPa, peak strain in compression $\varepsilon_{c1} = -0.20\%$, ultimate strain in compression $\varepsilon_{cu} = -0.35\%$, ultimate strain in tension $\varepsilon_{ctu} = 2f_c/E_{c0}$. The stress-strain diagram of steel is assumed elastic perfectly-plastic in both tension and compression. It is completely defined by the following quantities: yield strength $f_{sy} = 500$ MPa, elastic modulus $E_s = 210$ GPa, yield strain $\varepsilon_{sy} = f_{sy}/E_s$ and ultimate strain $\varepsilon_{cu} = 1.00\%$. Both the material strengths are in particular assumed as fuzzy variables $\mathbf{y} = [\, f_c \;\; f_{sy} \,]^T$ with symmetric triangular membership functions defined over the dimensionless interval bases [0.75 – 1.25] for concrete and [0.90 – 1.10] for steel. These supports are subdivided in 5×5=25 classes of strengths and three α−levels, progressively restricted over these classes as shown in Figure 2.a, are considered.

Besides the effect of gravity g (weight density $\gamma = 25$ kN/m^3), the bridge is subjected to the distribution of dead and live loads in Figure 1, with $g = 30$ kN/m, $q = 30$ kN/m, $Q = 600$ kN. Three different cross-sections, box-shaped and arranged as shown in Figure 1, are chosen.

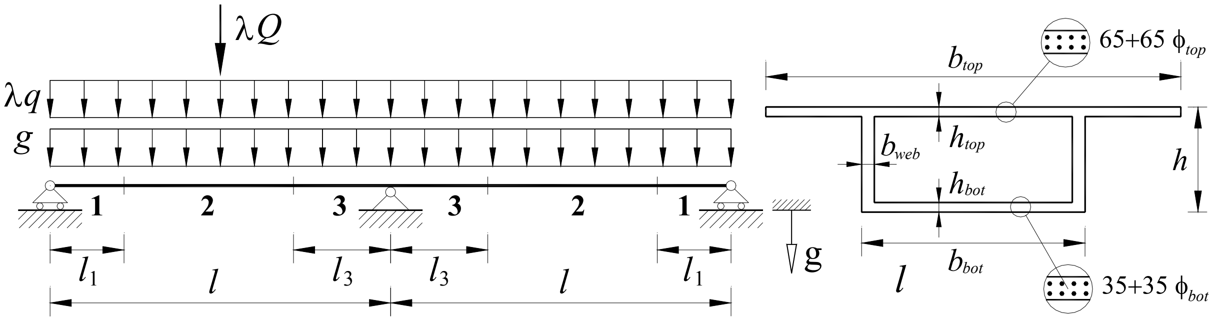


Figure 1: Continuous R.C. bridge: static scheme and cross-sectional geometry.

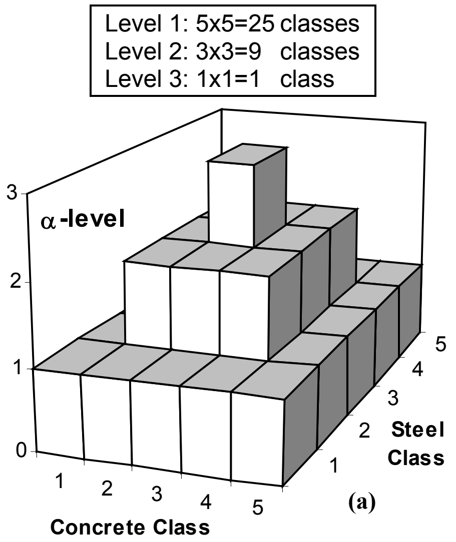
Each cross-section is reinforced with 200 bars, 65+65 located at the top flange and 35+35 at the bottom flange. The steel bars are placed at 50 mm from the nearest edge. For each section $i = 1,\ldots,3$, let: h^i_{top} and h^i_{bot} the thickness of the top and the bottom flange, respectively; h^i the total height of the cross-section; ϕ^i_{top} and ϕ^i_{bot} the diameter of the reinforcing bars at the top and the bottom, respectively. By assuming $h = h^1 = h^2 = h^3$, a total of $n = 15$ design variables $\mathbf{x} = [\,h \;\; h^1_{top} \;\; h^1_{bot} \;\; h^2_{top} \;\; h^2_{bot} \;\; h^3_{top} \;\; h^3_{bot} \;\; \phi^1_{top} \;\; \phi^1_{bot} \;\; \phi^2_{top} \;\; \phi^2_{bot} \;\; \phi^3_{top} \;\; \phi^3_{bot} \;\; l_1 \;\; l_3\,]^T$ are considered, being l_1 and l_3 the length of the parts of the bridge having section 1 and 3, respectively. Besides the side constraints listed in Table 1, the behavioral constraints $\lambda_S \geq 1$, $\lambda_U \geq 2$, $0.5\% \leq \rho_i \leq 6\%$ ($i = 1,\ldots,3$) are imposed, with the serviceability limit multiplier $\lambda_S(\mathbf{x})$ defined by assuming $\alpha_c = 0.45$, $\alpha_s = 0.80$, $s^+ = s^- = 75$ mm, and where $\rho_i(\mathbf{x})$ is the geometric ratio of the reinforcement of the section i. Finally, a unit cost ratio $c = 20$ is adopted.

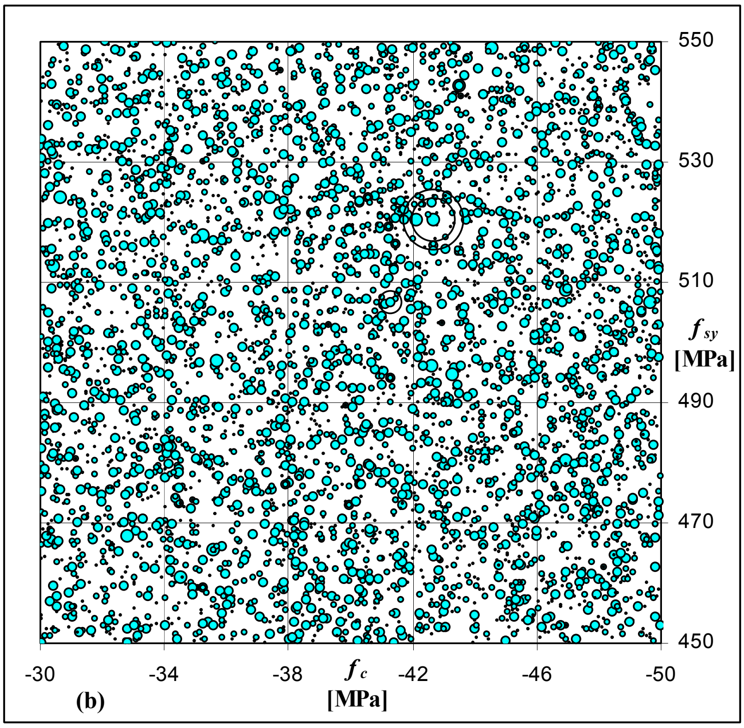
Figure 2.b shows the distribution of the simulated material strengths for the best current solution during 5000 generations of the genetic search, being the diameter of the shaded circles proportional to the fitness value of the corresponding solutions. In each of the 5×5=25 classes of material strengths the maximum fitness solution is certainly available. Thus, for each α–level, the maximum of the maximum solutions may be selected as the optimal solutions. However, from a safe design point of view, the minimum of the maximum solutions, identified by the blank circles in Figure 2.b, should be chosen. The values of the optimal design variables for the higher α–level are listed in Table 1, while the optimal fuzzy values of both the material strengths and the objective function for all the α–levels are listed in Figure 2.c.

Finally, as a concluding remark, it is worth noting that better solutions not necessarily deal with higher strengths of both the materials, nor with lower levels of design uncertainty.

x_i	h	h^1_{top}	h^1_{bot}	h^2_{top}	h^2_{bot}	h^3_{top}	h^3_{bot}	ϕ^1_{top}	ϕ^1_{bot}	ϕ^2_{top}	ϕ^2_{bot}	ϕ^3_{top}	ϕ^3_{bot}	l_1	l_3
x_i^-	1850	160	160	160	160	160	160	10	10	10	10	10	10	2700	2700
x_i^+	5000	780	780	780	780	780	780	24	24	24	24	24	24	12000	12000
x_{opt}	2500	180	160	180	420	300	200	24	24	22	18	24	14	6600	2700

Table 1. Side constraints and reliable optimal values of the design variables for the higher α–level ($\alpha \equiv \alpha_3$) [mm].





(c)	f_c [MPa]	f_{sy} [MPa]	$f(\mathbf{x}_{opt})$ [m^3]
α_1	-42.7	520.2	345.0
α_2	-42.7	520.2	345.0
α_3	-41.3	506.6	383.0

Figure 2. (a) Levels of uncertainty and classes of material strengths. (b) Distribution of the simulated strengths during the genetic search (the diameter of the shaded circles is proportional to the corresponding fitness values). The optimal solutions are identified by the blank circles (smaller circles for higher α–levels). (c) Optimal fuzzy values of the material strengths and the objective function for the more reliable optimal solutions.

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