



## EFFECTS OF SURFACE IRREGULARITIES ON THE DYNAMICAL RESPONSES OF BRIDGE-VEHICLE INTERACTION SYSTEMS

Yaxin Song<sup>1</sup>, Fabio Biondini<sup>2</sup>  
and Lawrence A. Bergman<sup>3</sup>, Member ASCE

### ABSTRACT

In a careful investigation of the effects of bridge surface roughness on the dynamical response of bridge-vehicle systems, three important aspects should be addressed. First, the interactions between bridge and vehicles resulting from coupling and irregularities on the bridge surface should be accounted for; second, a good representation of bridge surface roughness is necessary; and, finally, the model of the bridge should be as realistic as possible. However, in most of the available literature on the subject, one or more of these elements is downplayed. In this paper, a Monte Carlo simulation technique is employed to study the related random vibration problem. The irregularities in the bridge surface are modeled as a stationary, zero mean Gaussian process with a specified power spectral density (PSD), from which sample functions are generated by Shinozuka's method. To model a slab-girder bridge, a finite element formulation is used, in which the interaction dynamics between bridge and vehicles can be considered accurately. "Moving mass" and "moving oscillator" solutions are obtained. It is found that the "moving mass" simulation does not give reasonable results and that the distribution of Dynamic Amplification Factors (DAF) from the moving oscillator solution can be described well by a double log-normal probability distribution.

**Keywords:** Bridge-vehicle interaction, surface roughness, dynamical response analysis, Monte Carlo simulation.

### INTRODUCTION

The study of the effects of surface roughness on bridge and vehicle responses has been of interest to researchers for decades. To investigate the stochastic vibration problem, three important aspects should be addressed. First, the interaction between bridge and vehicles needs to be carefully taken into account. The effect of the vehicle loading on the bridge, referred to as the interaction force, can be affected significantly by the dynamic interaction between the bridge and vehicles, which results from coupling and bridge surface roughness. Second, a good representation of bridge surface roughness is necessary. Bridge surface irregularities exist because of imperfections in construction and wear. In essence, it is a random process. Finally, the model of the bridge should be as realistic as possible. For example, for a typical widely-used slab-girder bridge,

---

<sup>1</sup> Dept. of Aero. & Astro. Eng., Univ. of Illinois at Urbana-Champaign, Urbana, IL 61801. E-mail: ysong5@uiuc.edu

<sup>2</sup> Dept. of Structural Eng., Milan Polytechnic University, Milan 20133, Italy. E-mail: biondini@stru.polimi.it

<sup>3</sup> Dept. of Aero. & Astro. Eng., Univ. of Illinois at Urbana-Champaign, Urbana, IL 61801. E-mail: lbergman@uiuc.edu

a single beam or plate model may not be a good idealization. Unfortunately, many papers (Al-Khaleefi and Abdel-Rohman, 1999; Chompooming and Yener, 1995; Coussy *et al.*, 1989; Hwang and Nowak, 1991; Inbanathan and Wieland, 1987; Michaltsos and Konstantakopoulos, 2000; Palamas and Coussy, 1985) downplay one or more of these three aspects in their studies of bridge surface roughness effects.

This paper attempts to give a comprehensive analysis of the effects of bridge surface irregularities on dynamic responses of vehicle-bridge interaction systems in a more realistic way. We apply a Monte Carlo simulation technique for the stochastic vibration problem. To model the slab-girder bridge, a finite element formulation is used, in which the interaction between bridge and vehicles can be considered accurately. The sample functions of the bridge surface are generated from a given power spectral density function for road surface roughness on bridges by Shinozuka's method. Calculations are carried out for two vehicle models to simulate the so-called "moving mass" and "moving oscillator" problems. It is shown that the moving mass model fails to adequately predict the interaction effects and that the Dynamic Amplification Factor can be effectively described by a double log-normal distribution.

### GENERATION OF SAMPLES OF BRIDGE SURFACE FROM A GIVEN PSD

Analytical solutions for many problems in stochastic structural dynamics of practical importance can't be obtained. Monte Carlo simulation is often the only feasible approach. An important part of Monte Carlo simulation is the generation of sample functions of the underlying random processes in the problem; in our case, the generation of samples of irregularities in the bridge surface. Viewing the irregularities in the bridge surface as a stationary process with a zero mean value, we employed Shinozuka's method (Shinozuka and Deodatis, 1991; Shinozuka and Jan, 1972) to generate the sample functions from the given power spectral function for road surface roughness (Honda *et al.*, 1982).

According to Honda *et al.* (1982), the one-sided spectral density function  $G_r$  of the surface irregularities versus the spatial frequency  $f_r$  (in *cycle/m*) is approximated by

$$G_r(f_r) = \begin{cases} af_r^{-b} & \text{if } f_{r0} < f_r < f_{rN} \\ 0 & \text{if not} \end{cases} \quad (1)$$

where  $f_{r0}$  and  $f_{rN}$  are cut-off spatial frequencies and  $b = 1.94$  is a constant. Experimental values of  $a$  are given in different ranges to represent four conditions of pavement; i.e., "very good", "good", "ordinary" and "damaged". The one-sided power spectral density function expressed in terms of circular spatial frequency  $w_r = 2\pi f_r$  (*radian/m*) is

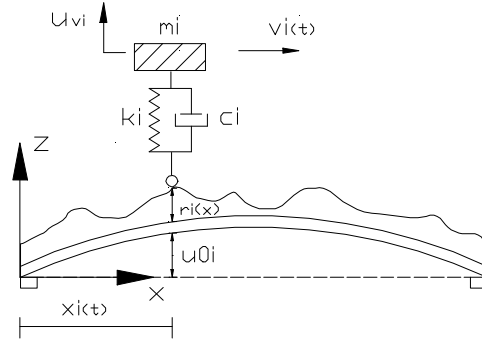
$$G_r(w_r) = G_r(f_r)/(2\pi) \quad (2)$$

Honda, et al. (1982) assumed that bridge surface roughness is a one variable process of the longitudinal coordinate of bridge. Using Shinozuka's method with the PSD of (2), samples of surface irregularities of the bridge  $r(x)$  are generated as

$$r(m \cdot \ddot{A}x) = \sqrt{2} \sum_{n=0}^{N-1} (G_r(w_{rn}) \ddot{A}w_r)^{1/2} \cos(w_{rn}m \cdot \ddot{A}x + \ddot{O}_n) \quad (3)$$

in which  $m = 0, 1, 2, \dots, M - 1$ ,  $n = 0, 1, 2, \dots, N - 1$ ;  $\ddot{A}w_r = w_{ru} / N$ ,  $w_{ru}$  is the cut-off circular spatial frequency;  $w_m = n\ddot{A}w_r$ ; and  $\ddot{O}_n$  are independent random phase angles distributed uniformly over the interval  $[0, 2\pi]$ . To avoid aliasing, we have  $M \geq 2N$ .

## FINITE ELEMENT FORMULATION



**FIG. 1. The schematic bridge-vehicle interaction system**

Fig. 1 shows a schematic of the bridge-vehicle interaction system. The finite element equations of motion of the bridge and vehicle  $i$  ( $i = 1, 2, \dots, n$ ) are

$$\mathbf{M}_b \ddot{\mathbf{U}}_b + \mathbf{C}_b \dot{\mathbf{U}}_b + \mathbf{K}_b \mathbf{U}_b = \sum_i \mathbf{N}_{c_i}^T \mathbf{F}_{c_i} \quad (4)$$

$$m_i \ddot{u}_{vi} + c_i (\dot{u}_{vi} - \dot{u}_{0i} - \dot{r}_i) + k_i (u_{vi} - u_{0i} - r_i) = 0 \quad (5)$$

in which  $\mathbf{M}_b$ ,  $\mathbf{C}_b$  and  $\mathbf{K}_b$  are mass, damping and stiffness matrices for the bridge;  $\ddot{\mathbf{U}}_b$ ,  $\dot{\mathbf{U}}_b$ ,  $\mathbf{U}_b$  are acceleration, velocity, displacement vectors of elemental nodes of the bridge;  $\ddot{u}_{vi}$ ,  $\dot{u}_{vi}$ ,  $u_{vi}$  are acceleration, velocity and displacement of vehicle  $i$ , and  $m_i$ ,  $k_i$ ,  $c_i$  are the mass, stiffness, damping of vehicle  $i$ , respectively. Also,  $\dot{u}_{0i}$  and  $u_{0i}$  are the velocity and displacement of the bridge at the contact position with vehicle  $i$ ;  $r_i = r(x_i)$  is the value of the surface irregularity function at the contact point;  $\mathbf{N}_{c_i}^T$  is the transpose vector of shape functions of bridge elements evaluated at the contact positions of vehicle  $i$ ; and  $\mathbf{F}_{c_i}$  is the contact force vector between the bridge and vehicle  $i$ . The contact force for vehicle  $i$  can be expressed as

$$F_{c_i} = -m_i g + c_i (\dot{u}_{vi} - \dot{u}_{0i} - \dot{r}_i) + k_i (u_{vi} - u_{0i} - r_i) = -m_i (g + \ddot{u}_{vi}) \quad (6)$$

From (4), (5), (6) and the constraint equations

$$u_{0i} = \mathbf{N}_{c_i} \mathbf{U}_b \quad (7)$$

we can get the global motion equation of the bridge-vehicle interaction system as

$$\begin{bmatrix} \mathbf{M}_b & \mathbf{M}_{bv} \\ \mathbf{0} & \mathbf{M}_v \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}}_b \\ \ddot{\mathbf{U}}_v \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_b & \mathbf{0} \\ \mathbf{C}_{vb} & \mathbf{C}_v \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}}_b \\ \dot{\mathbf{U}}_v \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_b & \mathbf{0} \\ \mathbf{K}_{vb} & \mathbf{K}_v \end{bmatrix} \begin{Bmatrix} \mathbf{U}_b \\ \mathbf{U}_v \end{Bmatrix} = \begin{Bmatrix} -\sum_i \mathbf{N}_{c_i}^T m_i g \\ \mathbf{F}_v \end{Bmatrix} \quad (8)$$

in which

$$\mathbf{M}_v = \text{diag}(m_1, m_2, \dots, m_{n_v}), \mathbf{C}_v = \text{diag}(c_1, c_2, \dots, c_{n_v}), \mathbf{K}_v = \text{diag}(k_1, k_2, \dots, k_{n_v}) \quad (9.a)$$

$$\ddot{\mathbf{U}}_v = \{\ddot{u}_{v1}, \ddot{u}_{v2}, \dots, \ddot{u}_{vn}\}^T, \dot{\mathbf{U}}_v = \{\dot{u}_{v1}, \dot{u}_{v2}, \dots, \dot{u}_{vn}\}^T, \mathbf{U}_v = \{u_{v1}, u_{v2}, \dots, u_{vn}\}^T \quad (9.b)$$

$$\mathbf{F}_v = \{c_1 \dot{r}_1 + k_1 r_1, c_2 \dot{r}_2 + k_2 r_2, \dots, c_n \dot{r}_n + k_n r_n\}^T \quad (9.c)$$

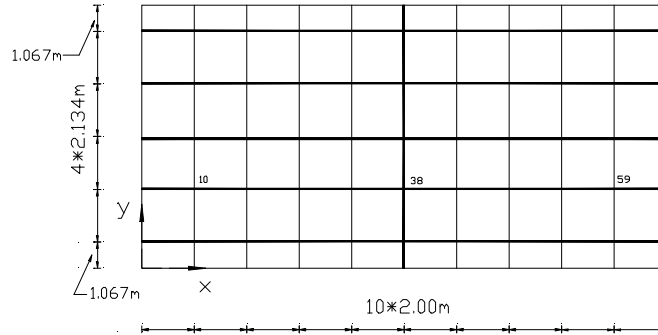
$$\mathbf{M}_{bv} = [\mathbf{N}_{c1}^T m_1, \mathbf{N}_{c2}^T m_2, \dots, \mathbf{N}_{cn}^T m_n] \quad (9.d)$$

$$\mathbf{C}_{vb} = [-\mathbf{N}_{c1}^T c_1, -\mathbf{N}_{c2}^T c_2, \dots, -\mathbf{N}_{cn}^T c_n]^T \quad (9.e)$$

$$\mathbf{K}_{vb} = [-\mathbf{N}_{c1,x}^T c_1 \dot{x}_1 - \mathbf{N}_{c1}^T k_1, -\mathbf{N}_{c2,x}^T c_2 \dot{x}_2 - \mathbf{N}_{c2}^T k_2, \dots, -\mathbf{N}_{cn,x}^T c_n \dot{x}_n - \mathbf{N}_{cn}^T k_n]^T \quad (9.f)$$

## NUMERICAL EXPERIMENTS

### Problem Description



**FIG. 2. Discretization of bridge with beam and rectangular plate elements (heavy lines represent beams)**

By Monte Carlo simulation, the effects of irregularities in the bridge surface are investigated for a slab-girder concrete bridge (refer to Wang *et al.* (1992) for the dimensions and material properties of the bridge). The slab is discretized with  $10 \times 6$  Kirchhoff rectangular plate elements ( $x \times y$  directions) and, correspondingly, 50 Euler beam elements parallel to the  $x$  direction and 6 Euler beam elements parallel to the  $y$  direction, as shown in Fig. 2. Rayleigh damping is considered, with the first two modal damping ratios  $V_1 = V_2 = 0.02$ . In total,  $n_s = 1000$  samples of bridge surface roughness are involved in the simulation. Two vehicle models are considered to address “moving mass” and “moving oscillator” problems, respectively. Both of them traverse the

bridge along the  $x$  direction ( $y_v=3.201\text{ m}$ ) with a velocity of  $v=10\text{m/s}$ . The parameters of the two vehicle models are: Vehicle I,  $m=1.0\times 10^4\text{ Kg}$ ,  $c=4.0\times 10^4\text{ Kg/s}$ ,  $k=1.0\times 10^8\text{ N/m}$ ; Vehicle II,  $m=1.0\times 10^4\text{ Kg}$ ,  $c=3.768\times 10^4\text{ Kg/s}$ ,  $k=3.944\times 10^5\text{ N/m}$ . Vehicle model I has a high natural frequency ( $100\text{ rad / s}$ ) to simulate the “moving mass” problem.

### Statistics of Dynamic Responses

Since the bridge surface roughness is modeled as a stationary zero mean stochastic process, the means of bridge displacement are coincident with deterministic results. By Monte Carlo simulation, we obtained the response standard deviations for normal and damaged pavements. Figures 3 and 4 show the time histories of the means of bridge displacement and their standard deviations (SD) at nodes 10, 38 and 59 for vehicle models I and II, respectively (refer to Fig. 2 for the locations of the nodes).

We can see from both figures that increased damage increases the RMS displacement responses. The responses for damaged pavement are larger than those for normal pavement. In the case of vehicle model I, the SD’s are initially oscillatory and become smoother later, while, for vehicle model II, the curves are smooth even at the beginning and give lower values than those for vehicle model I.

The Dynamic Amplification Factor (DAF) is defined as the ratio of the maximum absolute dynamic displacement response of the bridge considering surface irregularities to the corresponding static analysis result, at a certain point ( $x, y$ ). If we compute the DAF for each input realization, its histogram can be determined anywhere on the bridge, from which the pointwise distributions of the DAF can be estimated. The probability density functions (PDF) of the DAF’s at nodes 10, 38 and 59, for vehicle models I and II, are shown in Figures 5 and 6, respectively.

For both cases, it is clear that the DAF’s for damaged pavement have a wider distribution than those for normal pavement, and, by using a double log-normal PDF, the distribution of DAF’s can be described well. As to the differences, the DAF’s for vehicle model II concentrate in a much narrower area with smaller means (less than 1.25) than those for vehicle model I.

**TABLE 1. Means of DAF at Node 10, 38 and 59**

pavement	Node 10		Node 38		Node 59	
	Vehicle I	Vehicle II	Vehicle I	Vehicle II	Vehicle I	Vehicle II
Normal	4.25	1.12	1.78	1.10	3.58	1.13
Damaged	6.93	1.19	2.42	1.19	5.72	1.22

The means of DAF’s for “moving mass” (vehicle model I) and “moving oscillator” (vehicle model II) simulations are shown in Table 1. We observe that in the “moving mass” simulation, the means of DAF’s at locations near the ends of the bridge are much larger than those near the middle of the bridge, while in the “moving oscillator” simulation, the means of DAF’s are almost the same everywhere on the bridge. Since the irregularities of the bridge surface are assumed to form a stationary random process with zero mean value, the mean of DAF’s in our analysis should be the same as DAF’s obtained without considering bridge surface roughness. According to Barker and Puckett (1997), DAF’s take different values in different countries, and are given in terms of fundamental frequency of the bridge. For the bridge studied here (with a fundamental frequency of 3.45Hz), the DAF ranges from 1.08 to 1.8. Comparing the computed probability distribution of

DAF's for "moving mass" and "moving oscillator" solutions, it seems unlikely that the "moving mass" solution can lead to reasonable estimates of the effects of bridge irregularities upon bridge dynamics.

## **SUMMARY AND CONCLUSIONS**

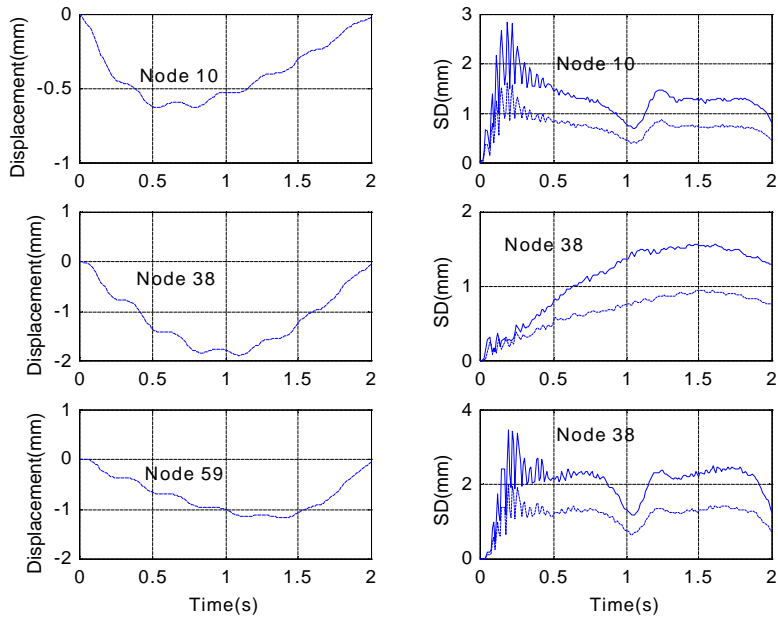
We noted that the irregularities in the bridge surface can have significant effects on the dynamic responses of a bridge and vehicles on it. We also noted that the "moving mass" simulation may not lead to reasonable estimates of the effects of bridge surface roughness upon bridge dynamics. Finally, we observed that the distribution of Dynamic Amplification Factor can be described well by the double log-normal probability function.

## **ACKNOWLEDGMENTS**

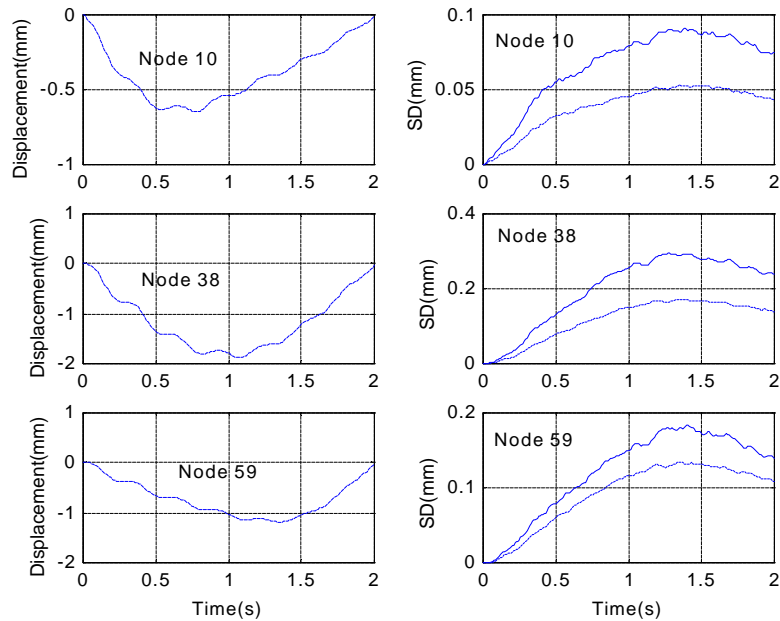
This work was supported in part by the National Science Foundation under grant CMS-9800136.

## **REFERENCES**

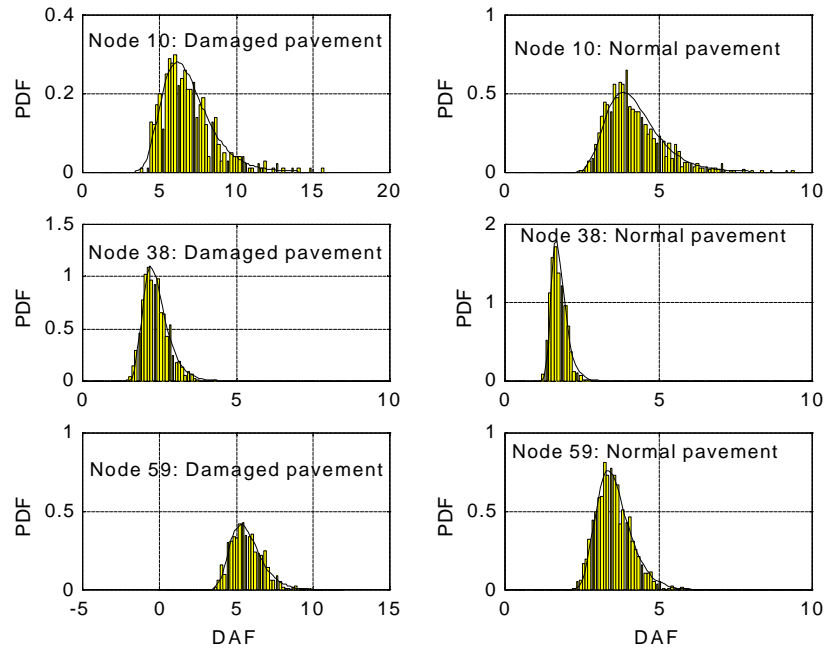
- Al-Khaleefi, A. M. and Abdel-Rohman, M., (1999), Effect of Humps on the Dynamic Response of Single-Span Bridges, *Journal of Vibration and Control*, **5**, 507-517.
- Barker, R. and Puckett, J. A., (1997), *Design of Highway Bridges: Based on AASHTO LRFD, Bridge Design Specifications*, John Wiley & Sons, INC..
- Chompooming, K. and Yener, M., (1995), The Influence of Roadway Surface Irregularities and Vehicle Deceleration on Bridge Dynamics Using the Method of Lines, *Journal of Sound and Vibration*, **183**(4), 567-589.
- Coussy, O., Said, M. and Hoove, J. P. van, (1989), The Influence of Random Surface Irregularities on the Dynamic Response of Bridges under Suspended Moving Loads, *Journal of Sound and Vibration*, **130**(6), 313-320.
- Hwang, E. and Nowak, A. S., (1991), Simulation of Dynamic Load for Bridges, *ASCE Journal of Structural Engineering*, **117**(5), 1413-1434.
- Honda, H., Kajikawa, Y. and Kobori, T., (1982), Spectral of Road Surface Roughness on Bridges, *ASCE Journal of the Structural Division*, **108**(9), 1956-1966.
- Inbanathan, M. J. and Wieland, M., (1987), Bridge Vibrations Due to Vehicle Moving over Rough Surface, *ASCE Journal of Structural Engineering*, **113**(9), 1994-2008.
- Michaltsos, G. T. and Konstantakopoulos, T. G., (2000), Dynamic Response of Bridge with Surface Deck Irregularities, *Journal of Vibration and Control*, **6**, 667-689.
- Palamas, J. and Coussy, O., (1985), Effects of Surface Irregularities upon the Dynamic Response of Bridges under Suspended Moving Loads, *Journal of Sound and vibration*, **99**(2), 235-245.
- Shinozuka, M. and Deodatis, G., (1991), Simulation of Stochastic Processes by Spectral Representation, *Applied Mechanics Reviews*, **44**(4), 191-203
- Shinozuka, M. and Jan, C-M., (1972), Digital Simulation of Random Processes and its Application, *Journal of Sound and Vibration*, **25**(1), 111-128.
- Wang, T. L., Huang, D. and Shahawy, M., (1992), Dynamic Response of Multigirder Bridges, *ASCE Journal of Structural Engineering*, **118**(8), 2222-2238.



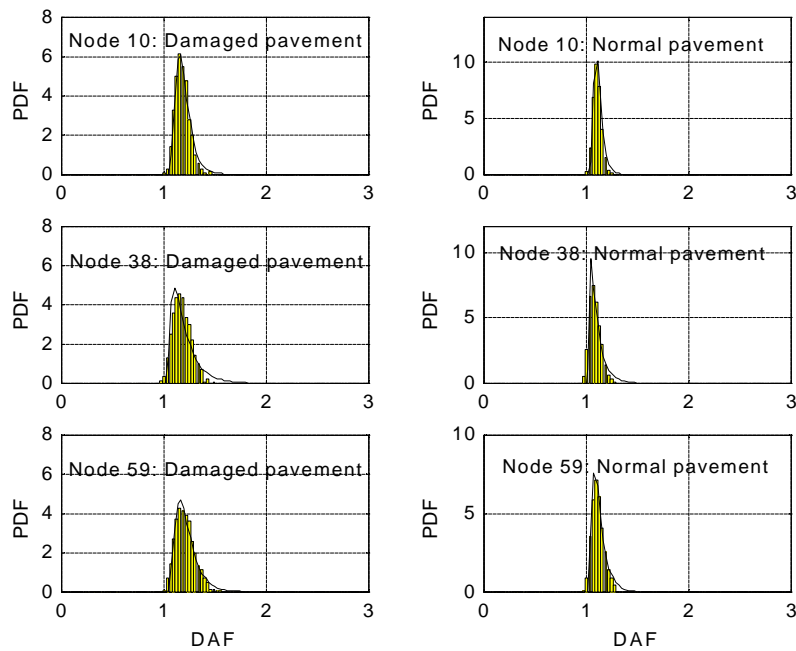
**FIG. 3. Average displacement responses and their SD's at nodes 10, 38 and 59 for vehicle Model I (Dotted lines: no irregularities considered; dashed lines: normal pavement; solid lines: damaged pavement)**



**FIG. 4. Average displacement responses and their SD's at nodes 10, 38 and 59 for vehicle Model II (Dotted lines: no irregularities considered; dashed lines: normal pavement; solid lines: damaged pavement)**



**FIG. 5. The distributions of DAF's and the fitted double lognormal distribution curves at nodes 10, 38 and 59 for vehicle Model I**



**FIG. 6. The distributions of DAF's and the fitted double lognormal distribution curves at nodes 10, 38 and 59 for vehicle Model II**