Reliability Assessment of Cable-Stayed Bridges

Fabio BIONDINI
PhD
Technical University of Milan
Milan, Italy

Franco BONTEMPI
Professor
University of Rome "La Sapienza"
Rome, Italy

Pier Giorgio MALERBA
Professor
University of Udine
Udine, Italy

Francesco MARTINEZ Y CABRERA
Professor
Technical University of Milan
Milan, Italy

Summary

The paper deals with the reliability assessment of P.C. cable-stayed bridges, but it is thought that the presented methodology is generally applicable. Due to several sources of uncertainties, the geometrical and mechanical properties which define the structural problem cannot be considered as deterministic quantities. In this work, such uncertainties are modelled by using a fuzzy criterion which considers the model parameters bounded between minimum and maximum suitable values. The reliability problem is formulated in terms of safety factor and the membership function over the failure interval is derived for several limit states by using a simulation technique. In particular, the strategic planning of the simulation is found by means of a genetic optimisation algorithm and the structural analyses are carried out by taking both material and geometrical non-linearity into account. An application to a cable-stayed bridge shows the effectiveness of the proposed procedure.

Keywords: cable-stayed bridges, structural design, structural reliability, uncertainty of the data, fuzzy criteria, non-linear analysis, simulation, genetic optimisation.

1. Introduction

Cable-stayed bridges exhibit a structural behaviour affected by several sources of non-linearity: the constitutive laws of the materials (concrete, normal and prestressing steels) and the geometrical effects induced by the change of the configuration and by the typical tension-hardening behaviour due to the change in sag of the stays. Consequently, the reliability assessment of this class of structure cannot be definitely assured without considering its actual non-linear behaviour. In this context, thought the reliability of the structure as resulting from a general and comprehensive investigation of all its failure modes, one must pay attention to the following aspects of the assessment process:
– *Available Data and Sources of Uncertainty.*
– *Limit States of Failure and Safety Factor.*
– *Structural Model and Non-Linear Analysis.*
– *Simulation Process and Synthesis of the Results*.

2. Available Data and Sources of Uncertainty

The methodology presented in the following has general validity and is here directly applied to the reliability analysis of a recent cable-stayed bridge under construction over Cujaba River in Brazil. The bridge, shown in Fig. 1, has been designed by Prof. Francesco Martinez y Cabrera with the collaboration of Dr. Eng. Emanuele Barbera. The contractor is Rivoli s.p.a. (Verona, Italy). The pylons of the bridge are cast-in-place, while the deck is completely precast and subdivided in 80 segments which are connected between them only by the shear friction assured by the prestressing forces. The design values of the material properties and of the initial prestressing in both the deck and the pylons are listed in Tab. 1. Besides the dead load g due to the self-weight of the structural and non structural members, a live load q=100 kN/m applied along the bridge deck is considered.

To the aim of the reliability analysis, the following 272 quantities are considered to be uncertain:
– the strength of the concrete in each segment of the deck and in each pylon (82 variables);
– the prestressing force in each cable (32 variables);
– the prestressing force in each stay (78 variables);
– the live load acting on each segment of the deck (80 variables).

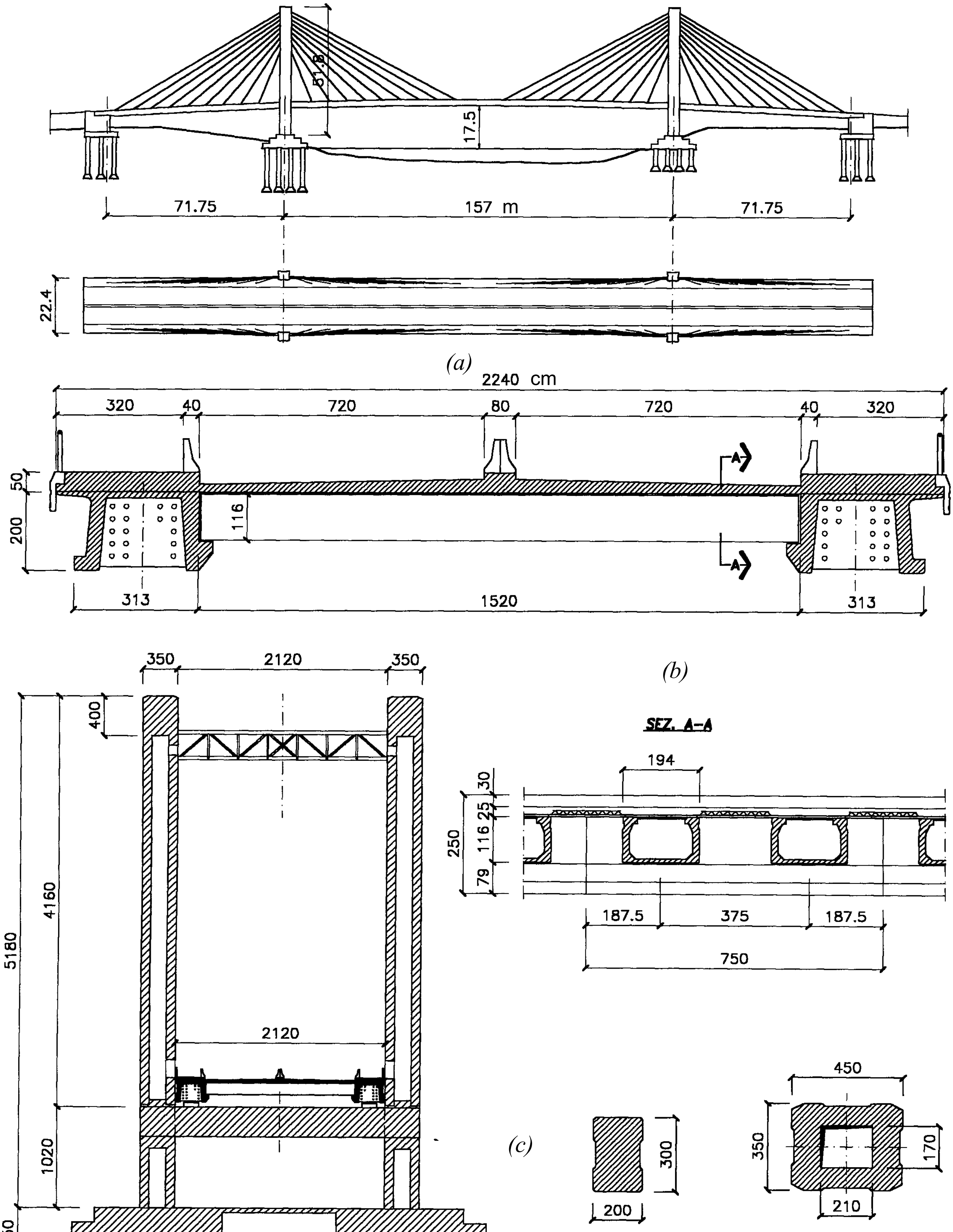


Figure 1. Cable-stayed bridge over Cujaba River in Brazil. (a) Longitudinal view and main geometrical dimensions of the bridge. Sectional views of (b) the deck and (c) the pylons.

Concrete Peak Strength (Deck)	f_c	–22.6 MPa
Concrete Peak Strength (Pylons)	f_c	–19.8 MPa
Concrete Peak Strain	ε_{c1}	–0.002
Concrete Ultimate Strain	ε_{cu}	–0.0035
Reinforcing Steel Strength	f_{sy}	383 MPa
Reinforcing Steel Young Modulus	E_s	205 GPa
Reinforcing Steel Ultimate Strain	ε_{su}	0.01
Prestressing Steel Strength	f_{py}	1617 MPa
Prestressing Steel Young Modulus	E_p	195 GPa
Prestressing Steel Ultimate Strain	ε_{pu}	0.015
Prestressing Stress (Deck)	σ_{po}	1000 MPa
Prestressing Stress (Stays)	σ_{po}	500 MPa

Table 1. Design values of the material properties and levels of prestressing.

Such 272 variables $\mathbf{x} = [x_1\ x_2\ \ldots\ x_{272}]^T$ are modelled by using a fuzzy criterion [3], [9]. The 190 variables which describe the concrete strengths and the prestressing forces are continuous with a triangular membership function having unit height for the nominal value x_{nom} and interval base $[0.80–1.20]x_{nom}$. The 80 variables associated to the live load are instead defined over the discrete set $[0; x_{nom}]$ with the same degree of membership for both the values, which means that the nominal load may be present or not on each segment of the deck with the same level of uncertainty.

3. Limit States of Failure and Safety Factor

The structural performances of the cable stayed bridge as regards the structural failure are described with reference to a specified set of limit states which separate desired states of the structure from undesired ones [5]. Based on the general concepts of the R.C. and P.C. design, when the strain in concrete ε_c, or in the reinforcing steel ε_s, or in the prestressing steel ε_p reaches the limit value ε_{cu}, ε_{su} or ε_{pu}, respectively, the collapse of the corresponding cross-section occurs. In addition, since the effectiveness of the connections between the segments which form the deck is depending on their level of compression, a limit state of decompression is assumed to be violated when the strain in concrete ε_c reaches a conventional strain limit in tension $\varepsilon_d = 0.0002$. An higher degree of failure of the connection is also considered to be verified when the amount of cracked area A_{cr} over a section reaches 1/3 of its total area A_c. Based on the same concept, also the slackness of the stays denote a failure condition which happens when the strain in the steel ε_p is no longer positive. However, the collapse of a single cross-section, a single deck connection or a single stay, doesn't necessarily lead to the collapse of the whole structure, the latter is caused by the loss of equilibrium arising when the reactions $\mathbf{r}$ requested for the loads $\mathbf{f}$ can no longer be developed.

Based on the previous considerations, the following seven limit states of failure have to be verified.

1. $-\varepsilon_c \le -\varepsilon_{cu}$ (1)

2. $|\varepsilon_s| \le \varepsilon_{su}$ (2)

3. $|\varepsilon_p| \le \varepsilon_{pu}$ (3)

4. $\varepsilon_c \le \varepsilon_d$ (only in the deck) (4)

5. $3A_{cr} \le A_c$ (only in the deck) (5)

6. $-\varepsilon_p \le 0$ (only in the stays) (6)

7. $\mathbf{f} \le \mathbf{r}$ (7)

These limit state functions $h(\mathbf{y}) \le 0$ refer to internal quantities of the system $\mathbf{y}$. Since the relationship $\mathbf{y} = \mathbf{y}(\mathbf{x})$ is generally available only in an implicit form, a check of the structural performance needs to be carried out at the load level. To this aim, it is useful to assume $\mathbf{f} = \mathbf{g} + \lambda\mathbf{q}$, where $\mathbf{g}$ is a vector of dead loads and $\mathbf{q}$ a vector of live loads whose intensity varies proportionally to a unique multiplier $\lambda \ge 0$. In this context, the limit load multiplier associated to the violation of a limit state of failure assumes the role of safety factor and the reliability of the structure against the nominal value of the loads has to be verified directly by checking the limit state condition $\lambda \ge 1$.

4. Structural Model and Non-Linear Analysis

The cable-stayed bridge is modelled as a two-dimensional framed structure.

The deck and the pylons are modelled by using a R.C./P.C. beam element whose formulation, based on the Bernoulli-Navier hypothesis, deals with both the mechanical non-linearity associated to the constitutive laws of the materials (concrete, ordinary and prestressing steels) and the geometrical non-linearity dues to the second order effects induced by the change of configuration of the beam [4], [6], [7]. In particular, both material $\mathbf{K}'_M$ and geometrical $\mathbf{K}'_G$ contributes to the element stiffness matrix $\mathbf{K}'$ and to the nodal forces vector $\mathbf{f}'$, equivalent to the applied loads $\mathbf{f}'_0$ and to the prestressing $\mathbf{f}'_p$, are derived by applying the principle of the virtual displacements and then evaluated by numerical integration over the length l of the beam:

$$\mathbf{K}' = \mathbf{K}'_M + \mathbf{K}'_G \qquad \mathbf{K}'_M = \int_0^l \mathbf{B}^T \mathbf{H} \mathbf{B} dx \qquad \mathbf{K}'_G = \int_0^l N\, \mathbf{G}^T \mathbf{G}\, dx \qquad \mathbf{f}' = \int_0^l \mathbf{N}^T (\mathbf{f}'_0 + \mathbf{f}'_p)\, dx \tag{8}$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{N}_a & \mathbf{0} \\ \mathbf{0} & \mathbf{N}_b \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \partial \mathbf{N}_a / \partial x & \mathbf{0} \\ \mathbf{0} & \partial^2 \mathbf{N}_b / \partial x^2 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} \mathbf{0} & \dfrac{\partial \mathbf{N}_b}{\partial x} \end{bmatrix} \tag{9}$$

where N is the axial force and $\mathbf{N}$ is a matrix of axial $\mathbf{N}_a$ and bending $\mathbf{N}_b$ displacement functions. In the following, the shape functions of a linear elastic beam element having uniform cross-sectional stiffness $\mathbf{H}$ and loaded only at its ends are adopted. However, due to material non-linearity, the cross-sectional stiffness distribution along the beam is non uniform even for prismatic members with uniform reinforcement. Thus, the matrix $\mathbf{H}$, as well as the sectional load vector equivalent to the prestressing $\mathbf{f}'_p$, have to be computed for each section by integration over the area of the composite element, or by assembling the contributes of concrete and steel. In particular, the contribute of the concrete is evaluated by subdividing the area of the beam cross section in four-nodes isoparametric sub-domains and by performing a Gauss-Legendre and/or a Gauss-Lobatto numerical integration over each sub-domain and then along the whole element.

The stays are instead modelled by using a truss element having axial stiffness only if its total strain is non negative. Besides the already mentioned mechanical and geometrical non-linearity, the formulation of such element takes also the characteristic tension-hardening behaviour due to the change in sag of the stays into account [7]. The contributes to the truss element stiffness matrix and to its nodal force vector are then derived in an analogous way like for the beam.

Finally, by assembling the stiffness matrix $\mathbf{K}$ and the vectors of the nodal forces $\mathbf{f}$ with respect to a global reference system, the equilibrium of the whole structure can be formally expressed as follows:

$$\mathbf{K}\mathbf{s} = \mathbf{f} \tag{10}$$

where $\mathbf{s}$ is the vector of the nodal displacements. It is worth noting that the vectors $\mathbf{f}$ and $\mathbf{s}$ have to be considered as total or incremental quantities depending on the nature of the stiffness matrix $\mathbf{K}=\mathbf{K}(\mathbf{s})$, or if a secant or a tangent formulation is adopted.

The stress-strain diagram of concrete is described by the Saenz's law in compression and by a no-strength model in tension. The stress-strain diagram of reinforcing steel is assumed elastic perfectly-plastic in tension and in compression. For prestressing steel the plastic branch is instead assumed as non-linear and described by a fifth degree polynomial function [5].

The two-dimensional model of the cable-stayed bridge is shown in Fig. 2, together with some of the results obtained by a step-by-step non-linear analysis based on the nominal value of the variables which define the structural system and by assuming the live load as uniformly distributed over the whole central span. Fig. 3 shows a typical subdivision adopted for the cross-section of both the deck and the pylons, as well as the corresponding distribution of both the reinforcing bars and the prestressing cables. Moreover, Fig. 4 shows the bending moment – axial force interaction curves of both the deck and the pylons associated to the limit state of sectional failure (limit states 1,2 and 3). In the same diagrams are also plotted the point loads corresponding to the evolution of the stress state in the Gauss sections of the structure for the loading case shown in Fig. 2. The limit values of the live load multiplier associated to several loading conditions are finally resumed in Tab.2.

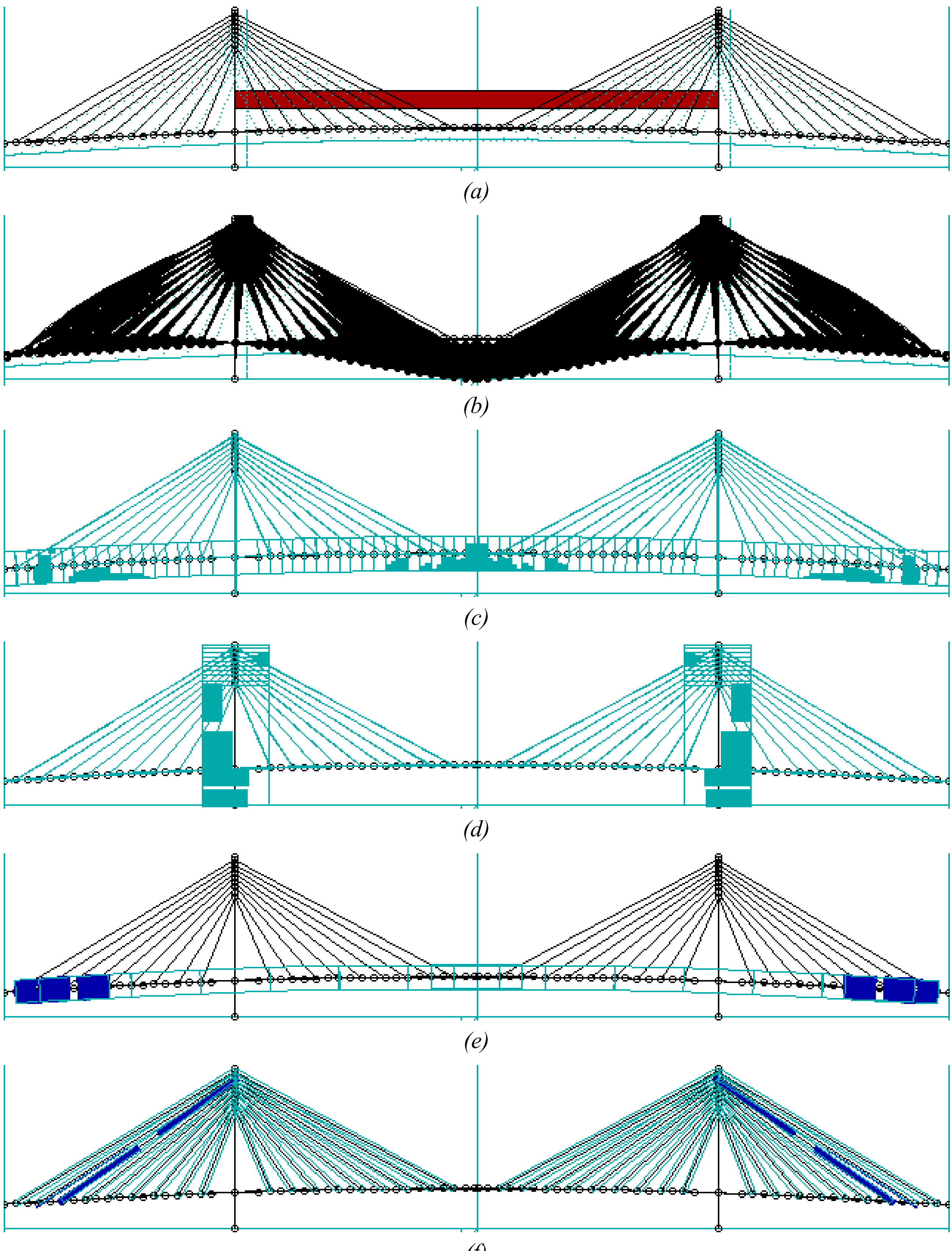
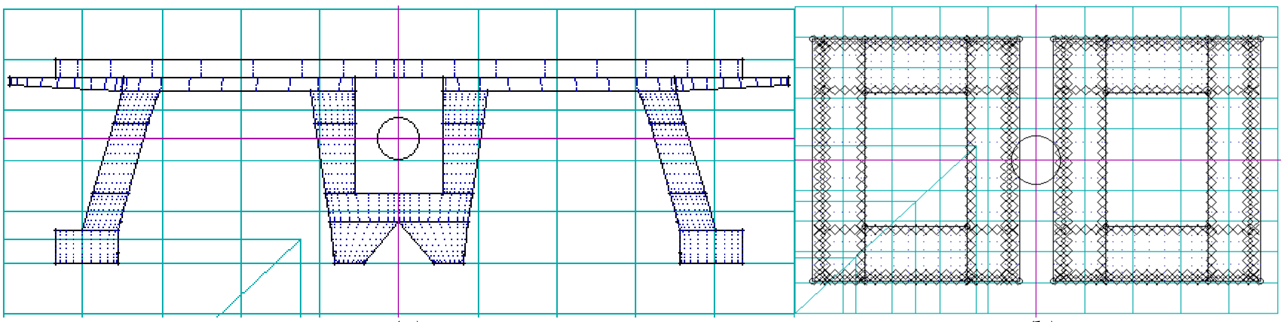
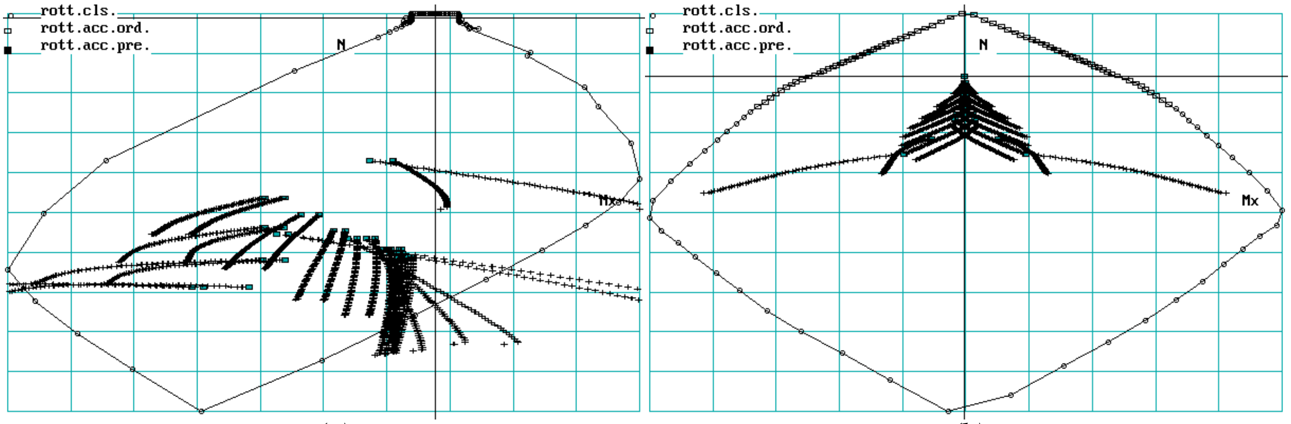


Figure 2. Step-by-step non-linear analysis of the cable-stayed bridge (nominal structure) under a given load condition. (a) Two-dimensional model of the bridge. (b) Evolution of the deformed shape. Distribution of cracking (shaded area) at collapse in both (c) the deck and (d) the pylons. Slackness in both (e) the segments of the deck and (f) the stays.



(a) *(b)*

Figure 3. Model of the typical cross-section of both (a) the deck and (b) the pylons. Subdivision of the concrete area and distribution of both the reinforcement bars and the prestressing cables.



(a) *(b)*

Figure 4. Bending moment (horizontal axis) – axial force (vertical axis) limit state curves for both (a) the deck and (b) the pylons. In the diagrams are also plotted the point loads corresponding to the evolution of the stress state in the Gauss sections of the structure for the loading case of Fig. 2.

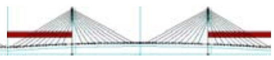
Load Condition – Limit State	**1**  λ_1	**2**  λ_2	**3**  λ_3	**4**  λ_4	λ_{min}
1	10	5.5	5.7	–	5.5
2	–	–	–	–	–
3	–	5.5	–	–	5.5
4	4.1	1.6	1.7	1.9	1.6
5	5.4	3.8	2.6	2.3	2.3
6	–	5.5	5.7	5.9	5.5
7	10	5.5	5.7	6.0	5.5

Table 2. Limit values of the live load multiplier for different load conditions (nominal structure).

5. Simulation Process and Synthesis of the Results

Based on the models previously introduced, a sample consisting of about 5000 simulations has been carried out for the α-level equal 0.5, which means that the continuous fuzzy variables have been considered as uniformly distributed over the range [0.90–1.10]. The membership functions of the limit load multiplier so obtained are presented in Fig. 5 and their distribution show that, for the nominal value of the live load (λ=1), the monitored limit states seems to be fully verified with the exception of the No.4. To this regard, since the prestressing action tends to balance the dead loads, it is worth noting that some minor localised violations of the decompression condition of the deck for λ=0 were expected and verified as not critical for the global reliability of the structure. Based on such considerations, the cable-stayed bridge appears to be safe with respect to the assumed α-level.

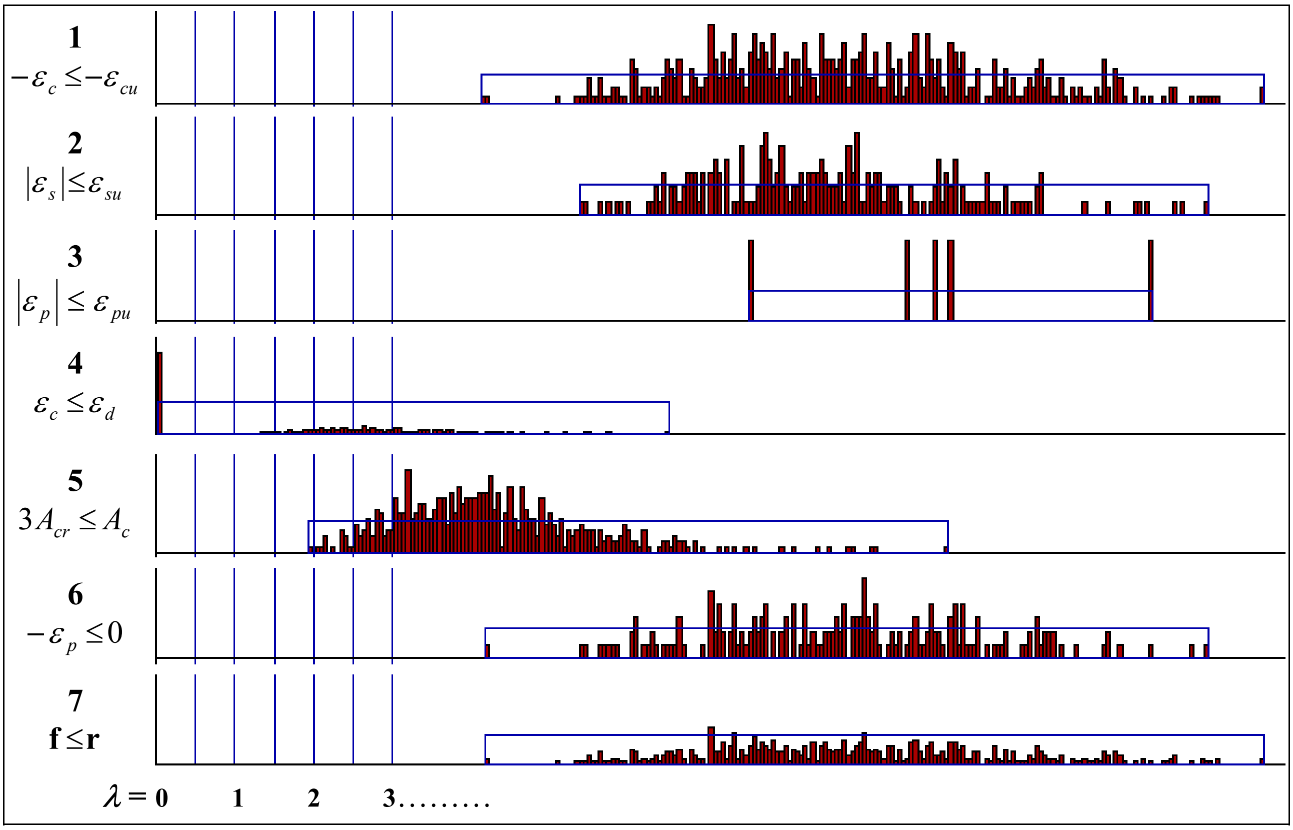


Figure 5. Membership functions of the load multiplier at failure for the α-level equal 0.5 (data sample of about 5000 simulations) derived by a random sampling method.

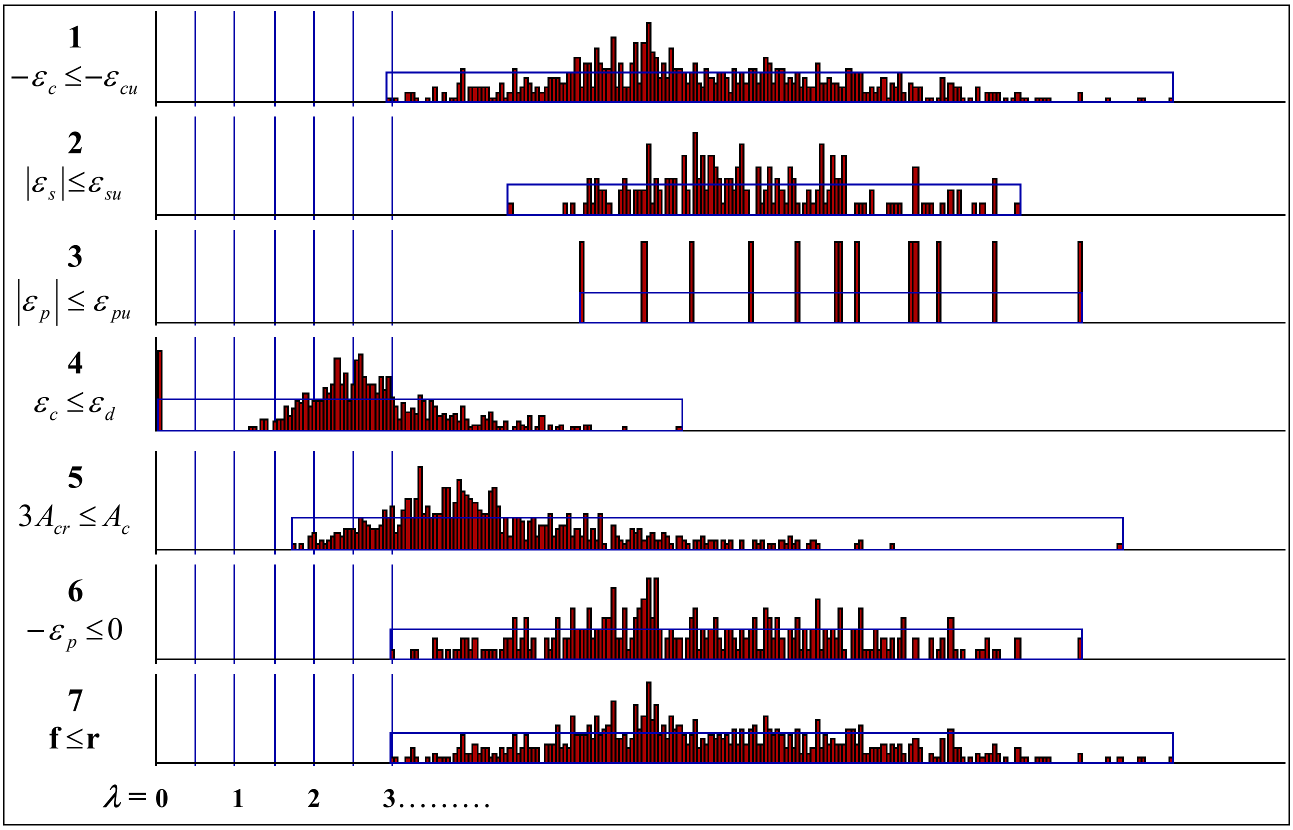


Figure 6. Membership functions of the load multiplier at failure for the α-level equal 0.5 (data sample of about 5000 simulations) derived by a genetically driven sampling method.

However, especially due to the high non-linearity of the structural problem, the validity of such result may be strongly dependent on the sample size and on the randomness of the sampling process. As a consequence, any guarantee is given about the achievement of the minimum values $\lambda_{\min}$.

A more refined simulation technique may be obtained by using an optimisation method to drive the sampling process [2]. In particular, by denoting with λ^+ a suitable upper bound of the minimum multiplier $\lambda_{\min}$, the following objective function to be maximised is introduced:

$$F(\mathbf{x}) = \sum_{i=1}^{n} \left(\lambda_i^+ - \lambda_{i,\min} \right) \tag{11}$$

being n=7 the number of limit states to be verified. A solution $\mathbf{x}$ of the optimisation problem which take the α-side constraints $\mathbf{x}^- \le \mathbf{x} \le \mathbf{x}^+$ into account is here found by means of a genetic algorithm [1].

As known, genetic algorithms are heuristic search techniques which belong to the class of stochastic algorithms, since they combine elements of deterministic and probabilistic search [8]. The search strategy works on a *population* $X = \{ \mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{x}_3 \ \ldots \ \mathbf{x}_m \}$ of *m individuals* $\mathbf{x}_k$ (k=1,2,…,m) subjected to an evolutionary process where individuals compete between them to survive in proportion to their *fitness* $F(\mathbf{x}) \ge 0$ with the *environment* $E = \{ \mathbf{x} \mid \mathbf{x}^- \le \mathbf{x} \le \mathbf{x}^+ \}$. In this process, population undergoes continuous reproduction by means of some *genetic operators* which, because of competition, tend to preserve best individuals. In the present case, a population size m=100 is assumed and the genetic search is performed until a total of about 5000 simulations is again obtained.

The effectiveness of the genetic algorithm in driving the simulation process can be appreciated from the inspection of Fig. 6, which highlight the higher capability of the genetic search in exploring the regions of the failure domain associated to the lowest values of the minimum load multipliers $\lambda_{\min}$. In fact, a comparison with the analogous results of Fig. 5 shows that the new samples are translated on the left along the horizontal axis, or towards more critical configurations for the structure. Since for the nominal live load (λ=1) the monitored limit states are not still violated, the structural safety of the cable-stayed bridge with respect to the assumed α-level can be considered definitively assured.

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